End Behavior and the Leading Term

The **end behavior** of a polynomial is a description of what happens as x becomes large in the positive or negative direction. To describe end behavior, we use the following notation:

$x \to \infty$	means	"x becomes large in the positive direction"

$x \to -\infty$	means	"x becomes large in the negative direction"

For example, the monomial $y = x^2$ has the following end behavior:

$$y \to \infty \text{ as } x \to -\infty \quad \text{and} \quad y \to \infty \text{ as } x \to \infty$$
$$\text{UP (left)} \quad \text{and} \quad \text{UP (right)}$$

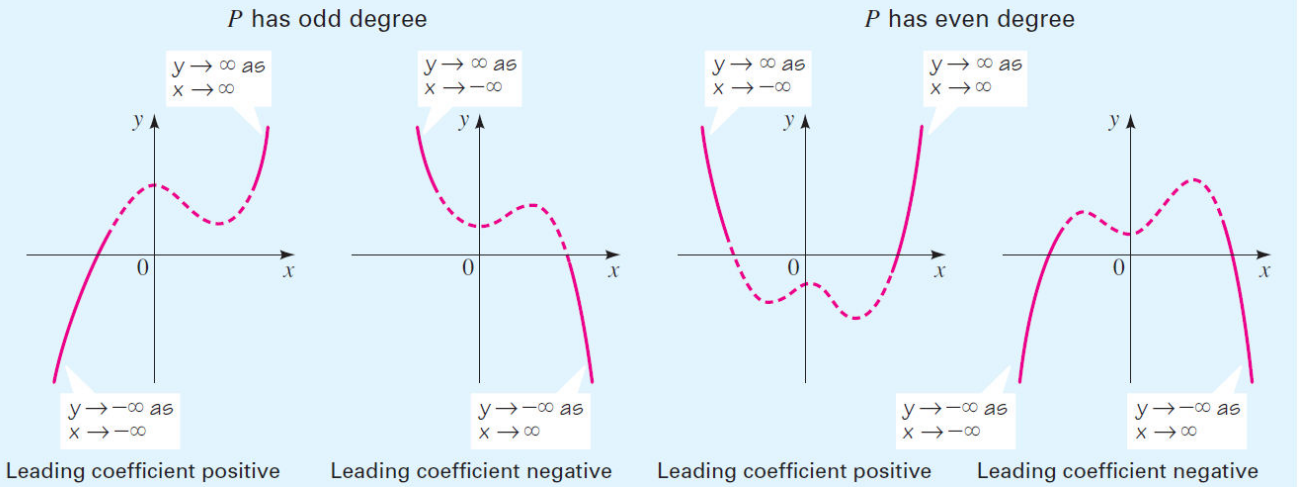
The monomial $y = x^3$ has the following end behavior:

$$y \to -\infty \text{ as } x \to -\infty \quad \text{and} \quad y \to \infty \text{ as } x \to \infty$$
$$\text{DOWN (left)} \quad \text{and} \quad \text{UP (right)}$$

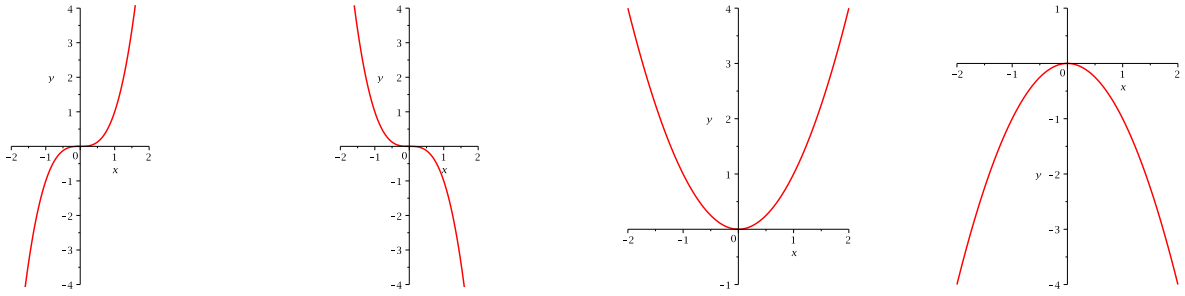
For any polynomial, *the end behavior is determined by the term that contains the highest power of* x, because when x is large, the other terms are relatively insignificant in size.

End Behavior of Polynomials

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.



COMPARE: Here are the graphs of the monomials x^3, $-x^3$, x^2, and $-x^2$.



EXAMPLE: Determine the end behavior of the polynomial

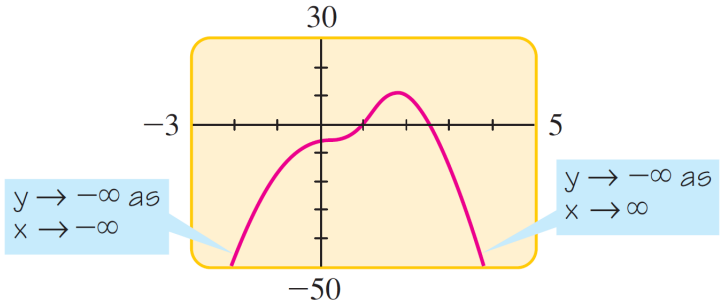
$$P(x) = -2x^4 + 5x^3 + 4x - 7$$

Solution: The polynomial P has degree 4 and leading coefficient -2. Thus, P has *even* degree and *negative* leading coefficient, so the end behavior of P is similar to $-x^2$:

$$y \to -\infty \text{ as } x \to -\infty \quad \text{and} \quad y \to -\infty \text{ as } x \to \infty$$

$$\text{DOWN (left)} \quad \text{and} \quad \text{DOWN (right)}$$

The graph in the Figure below illustrates the end behavior of P.



EXAMPLE: Determine the end behavior of the polynomial

$$P(x) = -3x^5 + 20x^2 + 60x + 2$$

Solution: The polynomial P has odd degree and negative leading coefficient. Thus, the end behavior of P is similar to $-x^3$:

$$y \to \infty \text{ as } x \to -\infty \quad \text{and} \quad y \to -\infty \text{ as } x \to \infty$$

$$\text{UP (left)} \quad \text{and} \quad \text{DOWN (right)}$$

EXAMPLE: Determine the end behavior of the polynomial

$$P(x) = 8x^7 - 7x^2 + 3x + 7$$

Solution: The polynomial P has odd degree and positive leading coefficient. Thus, the end behavior of P is similar to x^3:

$$y \to -\infty \text{ as } x \to -\infty \quad \text{and} \quad y \to \infty \text{ as } x \to \infty$$

$$\text{DOWN (left)} \quad \text{and} \quad \text{UP (right)}$$

EXAMPLE: Determine the end behavior of the polynomial

$$P(x) = 3x^5 - 5x^3 + 2x$$

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$$P(x) = 3x^5 - 5x^3 + 2x$$

Solution: The polynomial P has odd degree and positive leading coefficient. Thus, the end behavior of P is similar to x^3:

$$y \to -\infty \text{ as } x \to -\infty \quad \text{and} \quad y \to \infty \text{ as } x \to \infty$$

$$\text{DOWN (left)} \quad \text{and} \quad \text{UP (right)}$$