

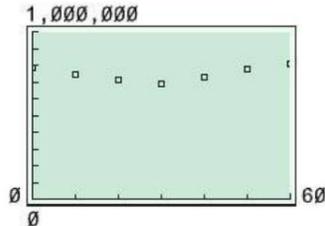
# Applications

EXAMPLE: The following table shows the population of the city of San Francisco, California in selected years.

Year	1950	1960	1970	1980	1990	2000	2010
Population	775,357	740,316	715,674	678,974	723,959	776,733	805,863

(a) Plot the data on a graphing calculator, with  $x = 0$  corresponding to the year 1950.

Solution: The points in the Figure below suggest the general shape of a fourth-degree (quartic) polynomial.

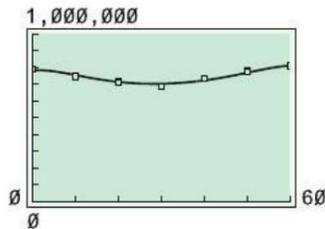


(b) Use quartic regression to obtain a model for these data.

Solution: The function obtained using quartic regression from a calculator or software is

$$f(x) = -.137x^4 + 16.07x^3 - 470.34x^2 + 542.65x + 773,944$$

Its graph, shown in the Figure below, appears to fit the data well.



(c) Use the model to estimate the population of San Francisco in the years 1985 and 2005.

Solution: The years 1985 and 2005 correspond to  $x = 35$  and  $x = 55$ , respectively. Verify that

$$f(35) \approx 700,186 \quad \text{and} \quad f(55) \approx 801,022$$

EXAMPLE: The following table shows the revenue and costs (in millions of dollars) for Ford Motor Company for the years 2004-2012. (Data from: www.morningstar.com.)

Year	Revenue	Costs
2004	171,652	168,165
2005	177,089	175,065
2006	160,123	172,736
2007	172,455	175,178
2008	146,277	160,949
2009	118,308	115,346
2010	128,954	122,397
2011	136,264	116,042
2012	134,252	128,588

(a) Let  $x = 4$  correspond to the year 2004. Use cubic regression to obtain models for the revenue data  $R(x)$  and the costs  $C(x)$ .

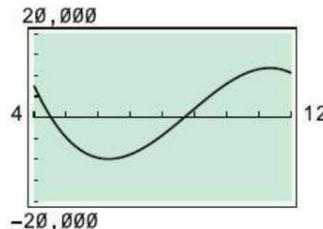
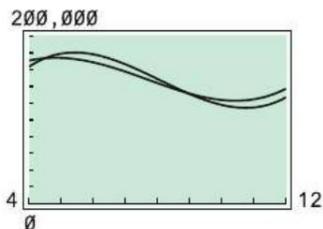
Solution: The functions obtained using cubic regression from a calculator or software are

$$R(x) = 551.1x^3 - 12,601x^2 + 82,828x + 7002$$

$$C(x) = 885.5x^3 - 21,438x^2 + 154,283x - 166,074$$

(b) Graph  $R(x)$  and  $C(x)$  on the same set of axes. Did costs ever exceed revenues?

Solution: The graph is shown in the Figure below (left). Since the lines cross twice, we can say that costs exceeded revenues in various periods.



(c) Find the profit function  $P(x)$  and show its graph.

Solution: The profit function is the difference between the revenue function and the cost function. We subtract the coefficients of the cost function from the respective coefficients of the revenue function.

$$\begin{aligned}
 P(x) &= R(x) - C(x) \\
 &= (551.1 - 885.5)x^3 + (-12,601 + 21,438)x^2 \\
 &\quad + (82,828 - 154,283)x + (7002 + 166,074) \\
 &= -334.4x^3 + 8837x^2 - 71,455x + 173,076.
 \end{aligned}$$

The graph of  $P(x)$  appears in the Figure above (right).

(d) According to the model of the profit function  $P(x)$ , in what years was Ford Motor Company profitable?

Solution: The motor company was profitable in 2004, 2009, 2010, 2011, and 2012 because that is where the graph is positive.