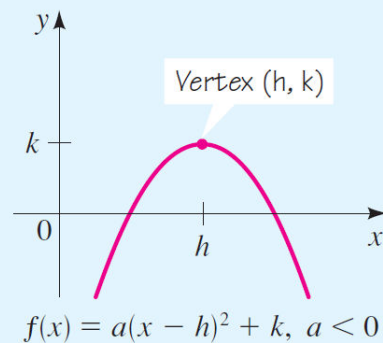
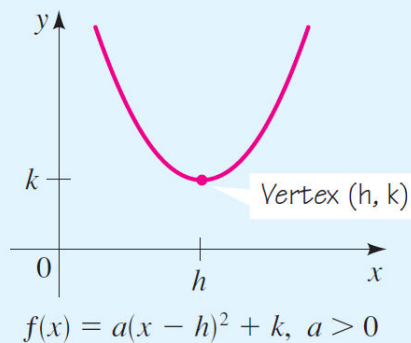


Standard Form of a Quadratic Function

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k) ; the parabola opens upward if $a > 0$ or downward if $a < 0$.



EXAMPLE: Let $f(x) = x^2 + 10x - 1$. Express f in standard form. Identify the vertex.

Solution: We have

$$\begin{aligned} f(x) &= x^2 + 10x - 1 \\ &= x^2 + 2x \cdot 5 - 1 \\ &= x^2 + 2x \cdot 5 + 5^2 - 5^2 - 1 \\ &= (x + 5)^2 - 26 \\ &= \left(x - (-5)\right)^2 + (-26) \end{aligned}$$

The vertex is $(-5, -26)$.

EXAMPLE: Let $f(x) = 2(x - 3)^2 + 5$.

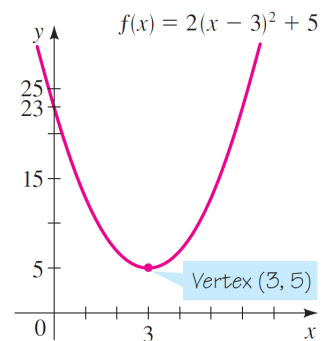
(a) Determine whether the given parabola opens upward or downward, and find its vertex.

(b) Sketch the graph of f .

Solution:

(a) Since $a = 2 > 0$, the parabola opens upward. The vertex of the parabola is at $(3, 5)$.

(b) The graph is shown on the right.



EXAMPLE: Let $f(x) = -(x + 1)^2 + 4$. Determine whether the given parabola opens upward or downward, and find its vertex. Then sketch the graph of f .

Solution: Since $a = -1 < 0$, the parabola opens downward. The vertex of the parabola is at $(-1, 4)$, because

$$f(x) = -(x + 1)^2 + 4 = -(x - (-1))^2 + 4 \implies h = -1, k = 4$$

The graph is shown below.

