Supply and Demand

The supply of and demand for an item are usually related to its price. Producers will supply large numbers of the item at a high price, but consumer demand will be low. As the price of the item decreases, consumer demand increases, but producers are less willing to supply large numbers of the item.

DEFINITION: The curves showing the quantity that will be supplied at a given price and the quantity that will be demanded at a given price are called **supply and demand curves**, respectively.

EXAMPLE: Joseph Nolan has studied the supply and demand for aluminum siding and has determined that the price per unit, p, and the quantity demanded, q, are related by the linear equation

$$p = 60 - \frac{3}{4}q$$

(a) Find the demand at a price of \$40 per unit.

Solution: Let $p = 40$. Then we have

$$\begin{aligned} p &= 60 - \frac{3}{4}q \\ 40 &= 60 - \frac{3}{4}q \\ 40{\color{blue}-60} &= 60 - \frac{3}{4}q{\color{blue}-60} \\ -20 &= -\frac{3}{4}q \\ -20 \cdot {\color{blue}\left(-\frac{4}{3}\right)} &= -\frac{3}{4}q \cdot {\color{blue}\left(-\frac{4}{3}\right)} \\ \frac{80}{3} &= q \end{aligned}$$

At a price of \$40 per unit, 80/3 (or $26\frac{2}{3}$) units will be demanded.

(b) Find the price if the demand is 32 units.

Solution: Let $q = 32$. Then we have

$$\begin{aligned} p &= 60 - \frac{3}{4}q \\ p &= 60 - \frac{3}{4}(32) \\ p &= 60 - 3(8) \\ p &= 60 - 24 \\ p &= 36 \end{aligned}$$

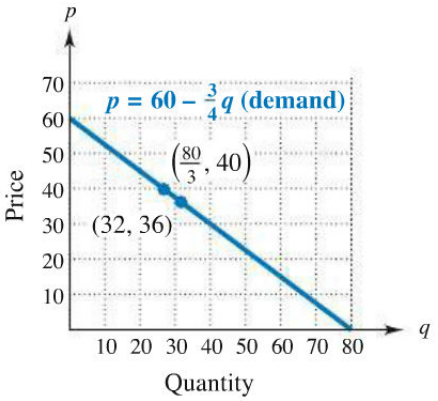
With a demand of 32 units, the price is \$36.

(c) Graph $p = 60 - \frac{3}{4}q$.

Solution: To graph $p = 60 - \frac{3}{4}q$, we can plug any two numbers in for x. For example, if $q = 0$, then

$$p = 60 - \frac{3}{4}q = 60 - \frac{3}{4}(0) = 60$$

Similarly, if $q = 32$, then $p = 36$ by (b). Using the points $(0, 60)$ and $(32, 36)$ yields the demand graph depicted in the Figure on the right. Only the portion of the graph in Quadrant I is shown, because supply and demand are meaningful only for positive values of p and q.



(d) From the Figure above, at a price of \$30, what quantity is demanded?

Solution: Price is located on the vertical axis. Look for 30 on the p-axis, and read across to where the line $p = 30$ crosses the demand graph. As the graph shows, this occurs where the quantity demanded is 40.

(e) At what price will 60 units be demanded?

Solution: Quantity is located on the horizontal axis. Find 60 on the q-axis, and read up to where the vertical line $q = 60$ crosses the demand graph. This occurs where the price is about \$15 per unit.

(f) What quantity is demanded at a price of \$60 per unit?

Solution: The point $(0, 60)$ on the demand graph shows that the demand is 0 at a price of \$60 (that is, there is no demand at such a high price).

EXAMPLE: Suppose the economist in the previous Example concludes that the supply q of siding is related to its price p by the equation

$$p = .85q$$

(a) Find the supply if the price is \$51 per unit.

Solution: We have

$$51 = .85q \quad \Longrightarrow \quad q = \frac{51}{.85} = 60$$

If the price is \$51 per unit, then 60 units will be supplied to the marketplace.

(b) Find the price per unit if the supply is 20 units.

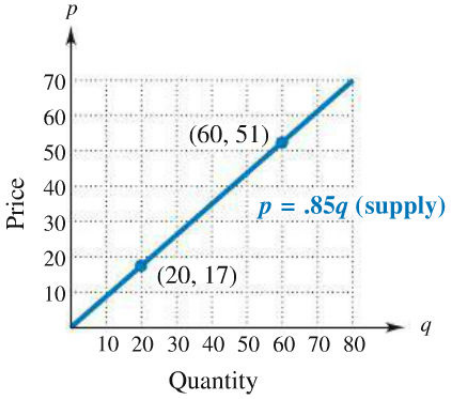
Solution: We have

$$p = .85(20) = 17$$

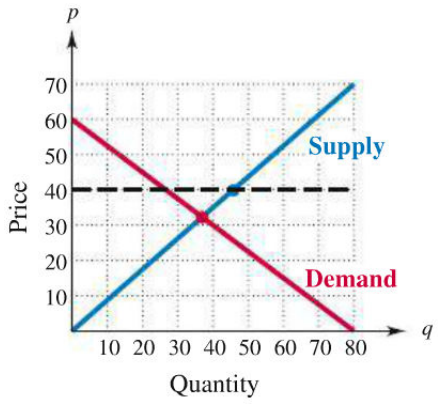
If the supply is 20 units, then the price is \$17 per unit.

(c) Graph the supply equation $p = .85q$.

Solution: Part (a) shows that the ordered pair $(60, 51)$ is on the graph of the supply equation, and part (b) shows that $(20, 17)$ is on the graph.



EXAMPLE: The supply and demand curves of the last two Examples are shown in the Figure below. Determine graphically whether there is a surplus or a shortage of supply at a price of \$40 per unit.



Solution: Find 40 on the vertical axis in the Figure above and read across to the point where the horizontal line $p = 40$ crosses the supply graph (that is, the point corresponding to a price of \$40). This point lies above the demand graph, so supply is greater than demand at a price of \$40, and there is a surplus of supply.

DEFINITION: The **equilibrium point** is the point where supply and demand are equal, that is, where the supply curve intersects the demand curve. Its second coordinate is the **equilibrium price**, the price at which the same quantity will be supplied as is demanded. Its first coordinate is the quantity that will be demanded and supplied at the equilibrium price; this number is called the **equilibrium quantity**.

EXAMPLE: In the situation described in the last three Examples, what is the equilibrium quantity? What is the equilibrium price?

Solution: The equilibrium point is where the supply and demand curves in the Figure above intersect. To find the quantity q at which the price given by the demand equation $p = 60 - .75q$ is the same as that given by the supply equation $p = .85q$, set these two expressions for p equal to each other and solve the resulting equation:

$$\begin{aligned} 60 - .75q &= .85q \\ 60 &= .85q + .75q \\ 60 &= 1.6q \\ q &= \frac{60}{1.6} = 37.5 \end{aligned}$$

Therefore, the equilibrium quantity is 37.5 units, the number of units for which supply will equal demand. Substituting $q = 37.5$ into either the demand or supply equation shows that

$$p = 60 - .75(37.5) = 31.875 \quad \text{or} \quad p = .85(37.5) = 31.875$$

to substitute into both equations, as we did here, to be sure that the same value of p results; if it does not, a mistake has been made.) In this case, the equilibrium point-the point whose coordinates are the equilibrium quantity and price — is $(37.5, 31.875)$.