Rates of Change

The rate of change of a linear function $f(x) = mx + b$ is the slope m

The value of a computer, or an automobile, or a machine *depreciates* (decreases) over time. **Linear depreciation** means that the value of the item at time x is given by a linear function $f(x) = mx + b$. The slope m of this line gives the rate of depreciation.

EXAMPLE: According to the *Kelley Blue Book*, a Ford Mustang two-door convertible that is worth \$14,776 today will be worth \$10,600 in two years (if it is in excellent condition with average mileage).

(a) Assuming linear depreciation, find the depreciation function for this car.

Solution: We know the car is worth \$14,776 now ($x = 0$) and will be worth \$10,600 in two years ($x = 2$). So the points $(0, 14{,}776)$ and $(2, 10{,}600)$ are on the graph of the linear depreciation function and can be used to determine its slope:

$$m = \frac{10,600 - 14,776}{2} = \frac{-4176}{2} = -2088$$

Using the point (0, 14,776), we find that the equation of the line is

$$y - 14,776 = -2088(x - 0)$$

$$y - 14,776 = -2088x$$

$$y = -2088x + 14,776.$$

Therefore, the rule of the depreciation function is $f(x) = -2088x + 14,776$.

(b) What will the car be worth in 4 years?

Solution: Evaluate f when $x = 4$:

$$f(x) = -2088x + 14,776$$

$$f(4) = -2088(4) + 14,776 = \$6424$$

(c) At what rate is the car depreciating?

Solution: The depreciation rate is given by the slope of $f(x) = -2088x + 14,776$, namely, -2088. This negative slope means that the car is decreasing in value an average of \$2088 a year.

DEFINITION: The rate of change of the cost function is called the **marginal cost**.

REMARK 1: Marginal cost is important to management in making decisions in such areas as cost control, pricing, and production planning. When the cost function is linear, say, $C(x) = mx + b$, the marginal cost is m (the *slope* of the graph of C).

REMARK 2: Marginal cost can also be thought of as the *cost of producing one more item*.

EXAMPLE: An electronics company manufactures handheld PCs. The cost function for one of its models is $C(x) = 160x + 750,000$.

(a) What are the fixed costs for this product?

Solution: The fixed costs are

$$C(0) = 160(0) + 750,000 = \$750,000$$

(b) What is the marginal cost?

Solution: The slope of $C(x) = 160x + 750,000$ is 160, so the marginal cost is \$160 per item.

(c) After 50,000 units have been produced, what is the cost of producing one more?

Solution: We have

$$C(50,000) = 160(50,000) + 750,000 = \$8,750,000 \quad \text{(cost of producing 50,000 units)}$$

$$C(50,001) = 160(50,001) + 750,000 = \$8,750,160 \quad \text{(cost of producing 50,001 units)}$$

The cost of the additional unit is the difference

$$C(50,001) - C(50,000) = 8,750,160 - 8,750,000 = \$160$$

Thus, the cost of one more item, is the marginal cost.

DEFINITION: The rate of change of a revenue function is called the **marginal revenue**.

REMARK: When the revenue function is linear, the marginal revenue is the *slope* of the line, as well as the *revenue from producing one more item*.

EXAMPLE: The energy company New York State Electric and Gas charges each residential customer a basic fee for electricity of \$15.11, plus \$.0333 per kilowatt hour (kWh).

(a) Assuming there are 700,000 residential customers, find the company's revenue function.

(b) What is the marginal revenue?

EXAMPLE: The energy company New York State Electric and Gas charges each residential customer a basic fee for electricity of \$15.11, plus \$.0333 per kilowatt hour (kWh).

(a) Assuming there are 700,000 residential customers, find the company's revenue function.

Solution: The monthly revenue from the basic fee is

$$15.11(700,000) = \$10,577,000$$

If x is the total number of kilowatt hours used by all customers, then the revenue from electricity use is $.0333x$. So the monthly revenue function is given by

$$R(x) = .0333x + 10,577,000$$

(b) What is the marginal revenue?

Solution: The marginal revenue (the rate at which revenue is changing) is given by the slope of the rate function: \$0.0333 per kWh.

The last two Examples are typical of the general case, as summarized here.

> In a **linear cost function** $C(x) = mx + b$, the marginal cost is m (the slope of the cost line) and the fixed cost is b (the y-intercept of the cost line). The marginal cost is the cost of producing one more item.
>
> Similarly, in a **linear revenue function** $R(x) = kx + d$, the marginal revenue is k (the slope of the revenue line), which is the revenue from selling one more item.