

Step Functions

The **greatest-integer function**, usually written $f(x) = [x]$, is defined by saying that $[x]$ denotes the largest integer that is less than or equal to x . For example,

$$[8] = 8, \quad [7.45] = 7, \quad [8.99] = 8, \quad [\pi] = 3, \quad [-1] = -1, \quad [-2.6] = -3, \quad [-5.1] = -6$$

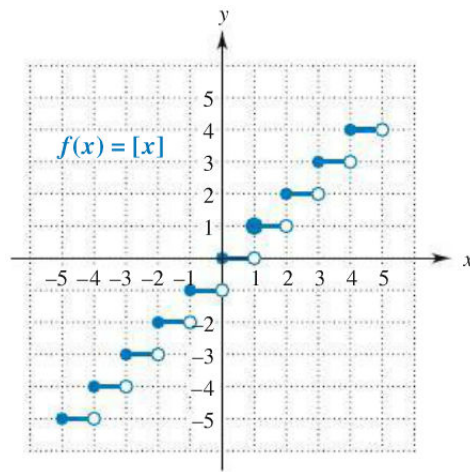
and so on.

EXAMPLE: Graph the greatest-integer function $f(x) = [x]$.

Solution: Consider the values of the function between each two consecutive integers — for instance,

x	$-2 \leq x < -1$	$-1 \leq x < 0$	$0 \leq x < 1$	$1 \leq x < 2$	$2 \leq x < 3$
$[x]$	-2	-1	0	1	2

Thus, between $x = -2$ and $x = -1$, the value of $f(x) = [x]$ is always -2 , so the graph there is a horizontal line segment, all of whose points have second coordinate -2 . The rest of the graph is obtained similarly (see the Figure below). An open circle in that figure indicates that the endpoint of the segment is *not* on the graph, whereas a closed circle indicates that the endpoint *is* on the graph.



Functions whose graphs resemble the graph of the greatest-integer function are sometimes called **step functions**.