EXAMPLES:

1. Let $f(x) = x + \sqrt{x}$. Then

$$f(0) = 0 + \sqrt{0} = 0$$
$$f(4) = 4 + \sqrt{4} = 6$$

2. Let $f(x) = 3x^2 + x - 5$. Then

$$f(-2) = 3 \cdot (-2)^2 + (-2) - 5 = 5$$
$$f(0) = 3 \cdot 0^2 + 0 - 5 = -5$$
$$f(4) = 3 \cdot 4^2 + 4 - 5 = 47$$
$$f\left(\frac{1}{2}\right) = 3 \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{2} - 5 = -\frac{15}{4}$$

3. Let $f(x) = \dfrac{\sqrt{x+1}}{x}$. Then

$$f(a-1) = \frac{\sqrt{a-1+1}}{a-1} = \frac{\sqrt{a}}{a-1}$$

REMARK: Note that

$$f(a-1) \neq \frac{\sqrt{a+1}}{a} - 1$$

4. Let $f(x) = 2x$. Then

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h) - 2x}{h} = \frac{2x + 2h - 2x}{h} = \frac{2h}{h} = 2$$

5. Let $f(x) = -5x + 7$. Then

$$\frac{f(x+h) - f(x)}{h} = \frac{\Big(-5(x+h) + 7\Big) - \Big(-5x + 7\Big)}{h} = \frac{-5x - 5h + 7 + 5x - 7}{h} = \frac{-5h}{h} = -5$$

6. Let $f(x) = -x^2 + 4x - 5$. Then

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{\Big(-(x+h)^2 + 4(x+h) - 5\Big) - \Big(-x^2 + 4x - 5\Big)}{h} \\
&= \frac{\Big(-x^2 - 2xh - h^2 + 4x + 4h - 5\Big) - \Big(-x^2 + 4x - 5\Big)}{h} \\
&= \frac{-x^2 - 2xh - h^2 + 4x + 4h - 5 + x^2 - 4x + 5}{h} \\
&= \frac{-2xh - h^2 + 4h}{h} = \frac{h(-2x - h + 4)}{h} = -2x - h + 4
\end{aligned}$$

REMARK: $\dfrac{f(x+h) - f(x)}{h}$ is called the **difference quotient** of the function f. Difference quotients are important in calculus.