

Section 3.1 Functions

DEFINITION: A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B . The set A is called the **domain** of f . The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain.

REMARK: In a function, each input produces a single output, but different inputs may produce the same output.

EXAMPLES: Which of the following rules describe functions?

(a) Use the optical reader at the checkout counter of the supermarket to convert codes to prices.

Solution: For each code, the reader produces exactly one price, so this is a function.

(b) The correspondence between a computer, x , and several users of the computer, y .

Solution: Since for a computer x there are several users y , this correspondence is not a function.

(c) Assign to each number x the number y given by this table:

x	1	1	2	2	3	3
y	3	-3	-5	-5	8	-8

Solution: Since $x = 1$ corresponds to more than one y -value (as does $x = 3$), this table does not define a function.

(d) Assign to each number x the number y given by the equation $y = 3x - 5$.

Solution: Because the equation determines a unique value of y for each value of x , it defines a function.

EXAMPLES: Decide whether each of the following equations defines y as a function of x .

(a) $y = x^2$

Solution: For a given value of x , calculating x^2 produces exactly one value of y . Because one value of the input variable leads to exactly one value of the output variable, $y = x^2$ defines y as a function of x .

(b) $x = y^2$

Solution: Suppose $x = 1$. Then $y^2 = x$ becomes $y^2 = 1$, from which $y = 1$ or $y = -1$. Since one value of x can lead to two values of y , $y^2 = x$ does not define y as a function of x .

(c) $x = y^3$

Solution: We have

$$x = y^3 \iff \sqrt[3]{x} = \sqrt[3]{y^3} \iff \sqrt[3]{x} = y$$

For a given value of x , calculating $\sqrt[3]{x}$ produces exactly one value of y . Because one value of the input variable leads to exactly one value of the output variable, $\sqrt[3]{x} = y$ (and therefore $x = y^3$) defines y as a function of x .