IMPORTANT FORMULAS (ALGEBRA)

1. Cancellation Property (Addition/Subtraction):

$$\boxed{A - B + B = A \quad \text{and} \quad A + B - B = A}$$

2. Cancellation Property (Multiplication/Division):

$$\boxed{\frac{A \cdot C}{B \cdot C} = \frac{A}{B} \qquad B, C \neq 0}$$

3. Distributive Property:

$$\boxed{A(B + C) = AB + AC \quad \text{and} \quad A(B - C) = AB - AC}$$

4. Important Identities:

(a) $A^2 - B^2 = (A - B)(A + B)$

(b) $(A + B)^2 = A^2 + 2AB + B^2$ $\qquad [(A + B)^2 \neq A^2 + B^2]$

(c) $(A - B)^2 = A^2 - 2AB + B^2$ $\qquad [(A - B)^2 \neq A^2 - B^2]$

(d) $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

(e) $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

5. Fractions:

(a) Multiplication:

$$\boxed{\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} \qquad B, D \neq 0} \qquad \left[\frac{A}{B} + \frac{C}{D} \neq \frac{A + C}{B + D}\right]$$

(b) Addition:

$$\boxed{\frac{A}{B} + \frac{C}{B} = \frac{A + C}{B} \quad \text{and} \quad \frac{A}{B} - \frac{C}{B} = \frac{A - C}{B} \qquad B \neq 0}$$

6. Powers $(A, B > 0)$:

(a) $A^0 = 1 \qquad A^{-n} = \dfrac{1}{A^n} \qquad A^{p/q} = \sqrt[q]{A^p} = (\sqrt[q]{A})^p$

(b) $A^n \cdot A^m = A^{n+m}$ $\qquad [A^n \cdot A^m \neq A^{n \cdot m}]$

(c) $\dfrac{A^n}{A^m} = A^{n-m}$ $\qquad \left[\dfrac{A^n}{A^m} \neq A^{n/m}\right]$

(d) $(A^n)^m = A^{n \cdot m}$

(e) $\left(\dfrac{A}{B}\right)^n = \dfrac{A^n}{B^n}, \quad (A \cdot B)^n = A^n \cdot B^n \quad \Rightarrow \quad \sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$ $\qquad \left[\sqrt{A + B} \neq \sqrt{A} + \sqrt{B}\right]$

IMPORTANT FORMULAS (PRECALCULUS)

LINES

Slope of a Line

The **slope** m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

Point-Slope Form of the Equation of a Line

An equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1)$$

Slope-Intercept Form of the Equation of a Line

An equation of the line that has slope m and y-intercept b is

$$y = mx + b$$

General Equation of a Line

The graph of every **linear equation**

$$Ax + By + C = 0 \qquad (A, B \text{ not both zero})$$

is a line. Conversely, every line is the graph of a linear equation.

Perpendicular Lines

Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$, that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

QUADRATIC FUNCTIONS

1. If $ax^2 + bx + c = 0$, then

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

2. The max or min value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$\boxed{x = -\frac{b}{2a}}$$

RATIONAL FUNCTIONS

Asymptotes of Rational Functions

Let r be the rational function

$$r(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

1. The vertical asymptotes of r are the lines $x = a$, where a is a zero of the denominator.

2. (a) If $n < m$, then r has horizontal asymptote $y = 0$.

(b) If $n = m$, then r has horizontal asymptote $y = \frac{a_n}{b_m}$.

(c) If $n > m$, then r has no horizontal asymptote.

LOGARITHMS

LAWS OF LOGARITHMS: If x and y are positive numbers, then

1. $\log_a(xy) = \log_a x + \log_a y$.
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$.
3. $\log_a(x^r) = r\log_a x$ where r is any real number.

CHANGE OF BASE FORMULA: For any positive a and b $(a, b \neq 1)$ we have

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

IMPORTANT FORMULAS (MATH FINANCE)

Simple Interest

The simple interest I on P dollars at a rate of interest r per year for t years is

$$\boldsymbol{I = Prt.}$$

Present Value for Simple Interest

The **present value** P of a future amount of A dollars at a simple interest rate r for t years is

$$\boldsymbol{P = \frac{A}{1 + rt}.}$$

Compound Interest

If P dollars are invested at interest rate i per period, then the **compound amount** (future value) A after n compounding periods is

$$A = P(1 + i)^n.$$

Continuous Compound Interest

The compound amount A for a deposit of P dollars at an interest rate r per year compounded continuously for t years is given by

$$A = Pe^{rt}.$$

Effective Rate (APY)

The effective rate (APY) corresponding to a stated rate of interest r compounded m times per year is

$$r_E = \left(1 + \frac{r}{m}\right)^m - 1.$$

Present Value for Compound Interest

The **present value** of A dollars compounded at an interest rate i per period for n periods is

$$\boldsymbol{P = \frac{A}{(1 + i)^n}, \qquad \text{or} \qquad P = A(1 + i)^{-n}.}$$

Future Value of an Ordinary Annuity

The future value S of an ordinary annuity used to accumulate funds is given by

$$\mathbf{S = R\left[\frac{(1+i)^n - 1}{i}\right]}, \quad \text{or} \quad \mathbf{S = R \cdot s_{\overline{n}|i}},$$

where

R is the payment at the end of each period,

i is the interest rate per period, and

n is the number of periods.

Future Value of an Annuity Due

The future value S of an annuity due used to accumulate funds is given by

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R$$

$$S = \begin{array}{c}\text{Future value of}\\ \text{an ordinary annuity}\\ \text{of } n+1 \text{ payments}\end{array} - \text{One payment,}$$

where

R is the payment at the beginning of each period,

i is the interest rate per period, and

n is the number of periods.

Present Value of an Ordinary Annuity

The present value P of an ordinary annuity is given by

$$\mathbf{P = R\left[\frac{1 - (1+i)^{-n}}{i}\right]}, \quad \text{or} \quad \mathbf{P = R \cdot a_{\overline{n}|i}},$$

where

R is the payment at the end of each period,

i is the interest rate per period, and

n is the number of periods.

Amortization Payments

A loan of P dollars at interest rate i per period may be amortized in n equal periodic payments of R dollars made at the end of each period, where

$$\mathbf{R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{Pi}{1-(1+i)^{-n}}}.$$

Remaining Balance

If a loan can be amortized by n payments of R dollars each at an interest rate i per period, then the *approximate* remaining balance B after x payments is

$$B = R\left[\frac{1 - (1 + i)^{-(n-x)}}{i}\right].$$

Present Value of an Annuity Due

The present value P of an annuity due is given by

$$P = R + R\left[\frac{1 - (1 + i)^{-(n-1)}}{i}\right],$$

$P =$ One payment $+$ Present value of an ordinary annuity of $n - 1$ payments

where

R is the payment at the beginning of each period,

i is the interest rate per period, and

n is the number of periods.

IMPORTANT FORMULAS (CALCULUS)

DEFINITION: The **average rate of change** of a function f as x changes from a to b is

$$\boxed{\frac{f(b)-f(a)}{b-a}}$$

The **instantaneous rate of change** for a function f when $x=a$ is

$$\boxed{\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}} \quad \text{or} \quad \boxed{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}$$

if these limits exist.

The **derivative of a function f at a number a** is

$$\boxed{f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}} \quad \text{or} \quad \boxed{f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}$$

if these limits exist.

THE CHAIN RULE: If f and g are both differentiable and $F=f\circ g$ is the composite function defined by $F(x)=f(g(x))$, then F is differentiable and F' is given by the product

$$\boxed{F'(x)=f'(g(x))\cdot g'(x)}$$

In Leibniz notation, if $y=f(u)$ and $u=g(x)$ are both differentiable functions, then

$$\boxed{\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}}$$

BASIC DIFFERENTIATION RULES

$c'=0, \quad x'=1$	$(u^n)'=nu^{n-1}\cdot u'$	$[cf(x)]'=cf'(x)$
$(a^u)'=a^u\ln a\cdot u'$	$(e^u)'=e^u\cdot u'$	$[f(x)\pm g(x)]'=f'(x)\pm g'(x)$
$(\log_a u)'=\dfrac{1}{u\ln a}\cdot u'$	$(\ln u)'=\dfrac{1}{u}\cdot u'$	$[f(x)g(x)]'=f'(x)g(x)+f(x)g'(x)$
		$\left[\dfrac{f(x)}{g(x)}\right]'=\dfrac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$

DEFINITION: A function F is called an **antiderivative** of f on an open interval I if

$$F'(x) = f(x) \text{ for all } x \text{ in } I$$

To denote the set of all antiderivatives of f on an open interval I we use the **indefinite integral** notation:

$$\boxed{\int f(x)dx = F(x) + C}$$

DEFINITION: If f is a continuous function defined on $[a, b]$, the **definite integral** of f from a to b is a number

$$\boxed{\int_a^b f(x)dx = \lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x]}$$

THEOREM (The Fundamental Theorem Of Calculus): If f is continuous on $[a, b]$, then

$$\boxed{\int_a^b f(x)dx = F(b) - F(a)}$$

where F is any antiderivative of f.

REMARK: A definite integral can be interpreted as a **net area,** that is, a difference of areas:

$$\int_a^b f(x)dx = A_1 - A_2$$

where A_1 is the area of the region above the x-axis and below the graph of f, and A_2 is the area of the region below the x-axis and above the graph of f.

THEOREM (Total Change in $F(x)$): Let f be a function such that f is continuous on the interval $[a, b]$ and $f(x) \geq 0$ for all x in $[a, b]$. If $f(x)$ is the rate of change of a function $F(x)$, then the **total change** in $F(x)$ as x goes from a to b is the area between the graph of $f(x)$ and the x-axis from $x = a$ to $x = b$.

THEOREM (Area between Two Curves): If f and g are continuous functions and $f(x) \geq g(x)$ on the interval $[a, b]$, then the area between the graphs of $f(x)$ and $g(x)$ from $x = a$ to $x = b$ is given by

$$\boxed{\int_a^b [f(x) - g(x)]dx}$$

BASIC INTEGRATION RULES

$\int cf(u)du = c\int f(u)du$	$\int [f(u) \pm g(u)]du = \int f(u)du \pm \int g(u)du$
$\int cdu = cu + C, \quad \int udu = \frac{u^2}{2} + C$	$\int_a^c f(u)du + \int_c^b f(u)du = \int_a^b f(u)du$
$\int u^n du = \frac{u^{n+1}}{n+1} + C \ (n \neq -1)$	$\int \frac{1}{u}du = \ln\|u\| + C$
$\int e^u du = e^u + C$	$\int e^{ku}du = \frac{1}{k}e^{ku} + C$

IMPORTANT FORMULAS (CALC FINANCE)

Marginal Cost

If $C(x)$ is the cost function, then the marginal cost (rate of change of cost) is given by the derivative $C'(x)$:

$$C'(x) \approx \text{ cost of making one more item after } x \text{ items have been made.}$$

The marginal revenue $R'(x)$ and marginal profit $P'(x)$ are interpreted similarly.

Average Cost

If the total cost of manufacturing x items is given by $C(x)$, then the **average cost per item** is

$$\overline{C}(x) = \frac{C(x)}{x}.$$

The **marginal average cost** is the derivative of the average-cost function $\overline{C}'(x)$.

Consumers' Surplus

If $D(q)$ is a demand function with equilibrium price p_0 and equilibrium demand q_0, then

$$\textbf{Consumers' surplus} = \int_0^{q_0} [D(q) - p_0]\, dq.$$

Producers' Surplus

If $S(q)$ is a supply function with equilibrium price p_0 and equilibrium supply q_0, then

$$\textbf{Producers' surplus} = \int_0^{q_0} [p_0 - S(q)]\, dq.$$

OPTIONAL FORMULAS (CALC FINANCE)

Elasticity of Demand

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

where q represents the quantity demanded and p the price of the item.

Marginal-Revenue Product

$$\frac{dR}{dn} = \left(p + x \cdot \frac{dp}{dx}\right) \frac{dx}{dn}$$

where R is total revenue from the daily production of x units, p is the price per unit, n is the number of employees.