1. Determine the end behavior of the polynomial \( P(x) = -x^7 + 27x + 18 \).

   \( \text{A} \) \( y \to \infty \) as \( x \to \infty \) and \( y \to \infty \) as \( x \to -\infty \)

   \( \text{B} \) \( y \to \infty \) as \( x \to \infty \) and \( y \to -\infty \) as \( x \to -\infty \)

   \( \text{C} \) \( y \to -\infty \) as \( x \to \infty \) and \( y \to \infty \) as \( x \to -\infty \)

   \( \text{D} \) \( y \to -\infty \) as \( x \to \infty \) and \( y \to -\infty \) as \( x \to -\infty \)

   \( \text{E} \) None of the above

Solution: The polynomial \( P \) has degree 7 and leading coefficient \(-1\). Thus, \( P \) has odd degree and negative leading coefficient, so it has the following end behavior:

\[ y \to -\infty \text{ as } x \to \infty \text{ and } y \to \infty \text{ as } x \to -\infty \]

2. Divide \( x^2 + x + 1 \) by \( x + 2 \).

   \( \text{A} \) quotient=\( x - 2 \), remainder=2

   \( \text{B} \) quotient=\( x - 3 \), remainder=1

   \( \text{C} \) quotient=\( x + 3 \), remainder=1

   \( \text{D} \) quotient=\( x - 1 \), remainder=3 \( \leftarrow \) Right Answer

   \( \text{E} \) quotient=\( x + 4 \), remainder=2

Solution: We have

\[
\begin{array}{c}
\begin{array}{c}
\underline{x + 2)} \quad x^2 + x + 1 \\
\hline
\quad x^2 + 2x \\
\quad -x^2 - 2x \\
\hline
\quad -x + 1 \\
\quad + x + 2 \\
\hline
\quad 3
\end{array}
\end{array}
\]
3. Factor the polynomial \( P(x) = 2x^4 - 6x^3 + 4x^2 \).

   \( A \) \( 2x^2(x + 1)(x - 2) \)

   \( B \) \( 2x^2(x - 1)(x - 2) \) \( \leftarrow \) Right Answer

   \( C \) \( 2x^2(x - 1)(x + 2) \)

   \( D \) \( 2x^2(x + 1)(x + 2) \)

   \( E \) None of the above

Solution: We have

\[
2x^4 - 6x^3 + 4x^2 = 2x^2(x^2 - 3x + 2) = 2x^2(x - 1)(x - 2)
\]

4. Find all rational zeros of the polynomial \( P(x) = x^4 - x^2 \).

   \( A \) \( 0 \)

   \( B \) \( 0, 1 \)

   \( C \) \( 0, -1 \)

   \( D \) \( 0, 1, -1 \) \( \leftarrow \) Right Answer

   \( E \) None of the above

Solution: We have

\[
x^4 - x^2 = x^2(x^2 - 1) = x^2(x - 1)(x + 1)
\]

therefore the polynomial \( P(x) = x^4 - x^2 \) has zeros 0, 1, -1.
In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. Let \( f(x) = 3x^2 - 6x + 2 \).
   (a) Express \( f \) in standard form using the technique of completing the square.
   Solution: We have
   \[
   f(x) = 3x^2 - 6x + 2 \\
   = 3(x^2 - 2x) + 2 \\
   = 3(x^2 - 2x \cdot 1) + 2 \\
   = 3(x^2 - 2x \cdot 1 + 1^2 - 1^2) + 2 \\
   = 3(x^2 - 2x \cdot 1 + 1^2) - 3 \cdot 1^2 + 2 \\
   = 3(x - 1)^2 - 1
   \]
   The standard form is \([f(x) = 3(x - 1)^2 - 1]\).
   
   (b) Sketch the graph of \( f \).
   Solution: The graph is a parabola that has its vertex at \((1, -1)\) and opens upward, as sketched in the Figure below.

   
   
   
   (c) Find the minimum value of \( f \).
   Solution: Since the coefficient of \( x^2 \) is positive, \( f \) has a minimum value. The minimum value is \( f(1) = -1 \).
2. A hockey team plays in an arena that has a seating capacity of 20,000 spectators. With the ticket price set at $10, average attendance at recent games has been 10,000. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000. Find the price that maximizes revenue from ticket sales.

Solution: The model we want is a function that gives the revenue for any ticket price. We know that

\[ \text{revenue} = \text{ticket price} \times \text{attendance} \]

There are two varying quantities: ticket price and attendance. Since the function we want depends on price, we let

\[ x = \text{ticket price} \]

Next, we must express the attendance in terms of \( x \).

The model is the function \( R \) that gives the revenue for a given ticket price \( x \).

\[ R(x) = x(10,000 - 1000x) \]
\[ R(x) = 10,000x - 1000x^2 \]

Since \( R(x) = 10,000x - 1000x^2 \) is a quadratic function with \( a = -1000 \) and \( b = 10,000 \), the maximum occurs at

\[ x = -\frac{b}{2a} = -\frac{10,000}{2(-1000)} = 5 \]

So a ticket price of \( \$5 \) yields the maximum revenue.