Final Exam

Algebra and Calculus

May 10, 2012

PLEASE READ THE FOLLOWING INFORMATION.

• This is a 110-minute exam. Calculators, books, notes, and other aids are not allowed.

• You may use the backs of the pages or the extra pages for scratch work. **Do not unstaple or remove pages as they can be lost in the grading process.**

• Please do not put your name on any page besides the first page.
MC I (12 points). This part consists of 12 multiple choice problems. Nothing more than the answer is required; consequently no partial credit will be awarded.

1. Evaluate $\frac{1/2}{1/4} - \frac{3}{5}$.
   - A 9/5
   - B 8/5
   - C 7/5
   - D 6/5
   - E None of the above

2. Evaluate $0.001 - \frac{0.01}{0.1}$.
   - A -1
   - B 1
   - C 2
   - D -2
   - E 0

3. Evaluate $-\frac{56}{7} + \frac{35}{5}$.
   - A 1
   - B -1
   - C -2
   - D 2
   - E 0

4. Evaluate $\sqrt[3]{27} - \sqrt{36} + \sqrt{16} - 1$.
   - A -2
   - B 0
   - C 2
   - D -1
   - E 1
5. Evaluate $\sqrt{0.01}$.
   A 1
   B 0.1
   C 0.01
   D 0.001
   E 0.0001

6. Evaluate $\csc\left(\frac{\pi}{6}\right) - \log_2 8$.
   A $-1$
   B 0
   C $-2$
   D 2
   E 1

7. Evaluate $\arccos 0$.
   A 1
   B 0
   C $\pi$
   D $\pi/2$
   E None of the above

8. Evaluate $\sqrt{9 + 16} - \frac{2 + 4}{2}$.
   A 7
   B 8
   C 9
   D 4
   E 2
9. Evaluate $\log 100$.
   A. 2
   B. 0
   C. 1
   D. 10
   E. None of the above

10. Evaluate $\sin 0 + \cos \pi$.
    A. $-1$
    B. 1
    C. 2
    D. $-2$
    E. 0

11. Evaluate $\tan \left( \frac{\pi}{3} \right)$.
    A. $\frac{1}{2}$
    B. $\sqrt{3}$
    C. $\frac{1}{\sqrt{3}}$
    D. 1
    E. None of the above

12. Evaluate $\sec^{-1}(-2)$.
    A. $\frac{\pi}{3}$
    B. $-\frac{\pi}{3}$
    C. $\frac{5\pi}{3}$
    D. $\frac{4\pi}{3}$
    E. $\frac{2\pi}{3}$
MC II (18 points). This part consists of 9 multiple choice problems. Nothing more than the answer is required; consequently no partial credit will be awarded.

1. $\cos(x + y)$ is equal to
   A. $\cos x + \cos y$
   B. $\cos x - \cos y$
   C. $\cos x \cos y - \sin x \sin y$
   D. $\cos x \cos y + \sin x \sin y$
   E. None of the above

2. If $f(x) = 6x - 18$, then $f\left(\frac{x}{3}\right)$ and $\frac{f(x)}{3}$ are
   A. $6x - 18, \ 6x - 3$
   B. $2x - 6, \ 2x - 18$
   C. $2x - 18, \ 2x - 3$
   D. $x - 3, \ x + 3$
   E. None of the above

3. The equation of the circle that has center $(3, -5)$ and radius 3 is
   A. $(x + 3)^2 + (x - 5)^2 = 3$
   B. $(x - 3)^2 + (x - 5)^2 = 9$
   C. $(x - 3)^2 + (x + 5)^2 = 9$
   D. $(x + 3)^2 + (x - 5)^2 = 9$
   E. None of the above
4. The vertical and horizontal asymptotes of \( \frac{6x^2 + 1}{2x^2 + x - 1} \) are

\[ A \quad x = \frac{1}{2}, \quad x = 1, \quad y = 2 \]

\[ B \quad x = \frac{1}{2}, \quad x = -1, \quad y = 3 \]

\[ C \quad x = \frac{1}{2}, \quad x = -1, \quad y = -3 \]

\[ D \quad x = -\frac{1}{2}, \quad x = -1, \quad y = 3 \]

\[ E \quad \text{None of the above} \]

5. The remainder of \( \frac{x^3 + x + 1}{x + 1} \) is

\[ A \quad 0 \]

\[ B \quad 1 \]

\[ C \quad -1 \]

\[ D \quad 2 \]

\[ E \quad -2 \]

6. The reference number for \( t = -\frac{5\pi}{6} \) is

\[ A \quad \frac{5\pi}{6} \]

\[ B \quad \frac{\sqrt{3}}{2} \]

\[ C \quad \frac{1}{2} \]

\[ D \quad -\frac{\pi}{6} \]

\[ E \quad \frac{\pi}{6} \]
7. The range of $|x| + 2$ is

A. $(-\infty, \infty)$
B. $(0, \infty)$
C. $[0, \infty)$
D. $(2, \infty)$
E. $[2, \infty)$

8. The phase amplitude ($|a|$), shift ($b$), and period ($p$) of $f(x) = 10 \cos\left(\frac{1}{2}x\right)$ are

A. $|a| = 10, \ b = \frac{1}{2}, \ p = 4\pi$
B. $|a| = -10, \ b = 2, \ p = 2\pi$
C. $|a| = 10, \ b = 0, \ p = \frac{\pi}{4}$
D. $|a| = 10, \ b = 0, \ p = \frac{\pi}{2}$
E. None of the above

9. A polynomial with degree 3 and zeros $-1, 1, 3$ is

A. $x^3 - 3x^2 - x + 3$
B. $x^3 + 3x^2 - x - 3$
C. $x^3 + x^2 - 5x + 3$
D. $x^4 - 6x^2 + 8x - 3$
E. None of the above
(60 points). Problems 1-9 are free response questions. You are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. (5 points) Factor $x^4 - x^3 - 2x^2 + 2x$.

2. (10 points) Solve the following inequalities:
   
   (a) $2|2x - 7| < 14$
   
   (b) $(x - 1)(x + 2) \leq -2$
3. (15 points) Solve the following equations:

(a) \( 2x - 1 = \sqrt{2 - x} \)

(b) \( \log_9(x - 5) + \log_9(x + 3) = 1 \)

(c) \( 1 + \sin x = 2 \cos^2 x \)
4. (5 points) Combine the expression \( \frac{1}{3} \log(x + 2) + \frac{1}{7} \log(x^2 + 1) - 5 \log(x^4 + 1) + \log x. \)

5. (5 points) Show that the equation \( x^2 + y^2 + 2x - 6y + 9 = 0 \) represents a circle, and find the radius and center of the circle.
6. (5 points) Plot \( f(x) = \frac{x - 2}{x - 1} \) using transformations.

7. (5 points) Let \( P(1, 2) \) and \( Q(2, -1) \) be two points on the coordinate plane. Find the slope of a line that is perpendicular to the line through \( P \) and \( Q \).
8. (5 points) Find the inverse function of \( f(x) = -\sqrt{x - 1} \).

9. (5 points) A manufacturer of soft drinks advertises their orange soda as “naturally flavored,” although it contains only 5% orange juice. A new federal regulation stipulates that to be called “natural” a drink must contain at least 10% fruit juice. How much pure orange juice must this manufacturer add to 900 gal of orange soda to conform to the new regulation?