Final Exam

Algebra and Calculus

May 10, 2012

PLEASE READ THE FOLLOWING INFORMATION.

• This is a \textbf{110-minute} exam. Calculators, books, notes, and other aids are not allowed.

• You may use the backs of the pages or the extra pages for scratch work. \textbf{Do not unstaple or remove pages as they can be lost in the grading process.}

• Please do not put your name on any page besides the first page.
MC I (12 points). This part consists of 12 multiple choice problems. Nothing more than the answer is required; consequently no partial credit will be awarded.

1. Evaluate $\frac{1/2}{1/4} - 3/5$.

A 9/5
B 8/5
C 7/5  (CORRECT ANSWER)
D 6/5
E None of the above

2. Evaluate $\frac{0.001}{0.01} - \frac{0.01}{0.1}$.

A -1
B 1
C 2
D -2
E 0  (CORRECT ANSWER)

3. Evaluate $-\frac{56}{7} + \frac{35}{5}$.

A 1
B -1  (CORRECT ANSWER)
C -2
D 2
E 0

4. Evaluate $\sqrt{27} - \sqrt{36} + \sqrt{16} - 1$.

A -2  (CORRECT ANSWER)
B 0
C 2
D -1
E 1
5. Evaluate $\sqrt{0.01}$.
   \[\begin{align*}
   \text{A} & \quad 1 \\
   \text{B} & \quad 0.1 \quad \text{(CORRECT ANSWER)} \\
   \text{C} & \quad 0.01 \\
   \text{D} & \quad 0.001 \\
   \text{E} & \quad 0.0001
   \end{align*}\]

6. Evaluate $\csc\left(\frac{\pi}{6}\right) - \log_2 8$.
   \[\begin{align*}
   \text{A} & \quad -1 \quad \text{(CORRECT ANSWER)} \\
   \text{B} & \quad 0 \\
   \text{C} & \quad -2 \\
   \text{D} & \quad 2 \\
   \text{E} & \quad 1
   \end{align*}\]

7. Evaluate $\arccos 0$.
   \[\begin{align*}
   \text{A} & \quad 1 \\
   \text{B} & \quad 0 \\
   \text{C} & \quad \pi \\
   \text{D} & \quad \frac{\pi}{2} \quad \text{(CORRECT ANSWER)} \\
   \text{E} & \quad \text{None of the above}
   \end{align*}\]

8. Evaluate $\sqrt{9 + 16} - \frac{2 + 4}{2}$.
   \[\begin{align*}
   \text{A} & \quad 7 \\
   \text{B} & \quad 8 \\
   \text{C} & \quad 9 \\
   \text{D} & \quad 4 \\
   \text{E} & \quad 2 \quad \text{(CORRECT ANSWER)}
   \end{align*}\]
9. Evaluate \( \log 100 \).
   - A 2 (CORRECT ANSWER)
   - B 0
   - C 1
   - D 10
   - E None of the above

10. Evaluate \( \sin 0 + \cos \pi \).
    - A \(-1\) (CORRECT ANSWER)
    - B 1
    - C 2
    - D \(-2\)
    - E 0

11. Evaluate \( \tan \left( \frac{\pi}{3} \right) \).
    - A \(\frac{1}{2}\)
    - B \(\sqrt{3}\) (CORRECT ANSWER)
    - C \(\frac{1}{\sqrt{3}}\)
    - D 1
    - E None of the above

12. Evaluate \( \sec^{-1}(-2) \).
    - A \(\frac{\pi}{3}\)
    - B \(-\frac{\pi}{3}\)
    - C \(\frac{5\pi}{3}\)
    - D \(\frac{4\pi}{3}\) (CORRECT ANSWER)
    - E \(\frac{2\pi}{3}\)
MC II (18 points). This part consists of 9 multiple choice problems. Nothing more than the answer is required; consequently no partial credit will be awarded.

1. \( \cos(x + y) \) is equal to
   \[ \begin{array}{ll}
   \text{A} & \cos x + \cos y \\
   \text{B} & \cos x - \cos y \\
   \text{C} & \cos x \cos y - \sin x \sin y \quad \text{(CORRECT ANSWER)} \\
   \text{D} & \cos x \cos y + \sin x \sin y \\
   \text{E} & \text{None of the above}
   \end{array} \]

2. If \( f(x) = 6x - 18 \), then \( f\left(\frac{x}{3}\right) \) and \( \frac{f(x)}{3} \) are
   \[ \begin{array}{ll}
   \text{A} & 6x - 18, \ 6x - 3 \\
   \text{B} & 2x - 6, \ 2x - 18 \\
   \text{C} & 2x - 18, \ 2x - 3 \\
   \text{D} & x - 3, \ x + 3 \\
   \text{E} & \text{None of the above} \quad \text{(CORRECT ANSWER)}
   \end{array} \]

3. The equation of the circle that has center \((3, -5)\) and radius 3 is
   \[ \begin{array}{ll}
   \text{A} & (x + 3)^2 + (x - 5)^2 = 3 \\
   \text{B} & (x - 3)^2 + (x - 5)^2 = 9 \\
   \text{C} & (x - 3)^2 + (x + 5)^2 = 9 \quad \text{(CORRECT ANSWER)} \\
   \text{D} & (x + 3)^2 + (x - 5)^2 = 9 \\
   \text{E} & \text{None of the above}
   \end{array} \]
4. The vertical and horizontal asymptotes of \( \frac{6x^2 + 1}{2x^2 + x - 1} \) are

A. \( x = \frac{1}{2}, x = 1, y = 2 \)

B. \( x = \frac{1}{2}, x = -1, y = 3 \)  \( \text{(CORRECT ANSWER)} \)

C. \( x = \frac{1}{2}, x = -1, y = -3 \)

D. \( x = -\frac{1}{2}, x = -1, y = 3 \)

E. None of the above

5. The remainder of \( \frac{x^3 + x + 1}{x + 1} \) is

A. 0

B. 1

C. -1  \( \text{(CORRECT ANSWER)} \)

D. 2

E. -2

6. The reference number for \( t = -\frac{5\pi}{6} \) is

A. \( \frac{5\pi}{6} \)

B. \( \frac{\sqrt{3}}{2} \)

C. \( \frac{1}{2} \)

D. \( -\frac{\pi}{6} \)

E. \( \frac{\pi}{6} \)  \( \text{(CORRECT ANSWER)} \)
7. The range of $|x| + 2$ is
   - A $(-\infty, \infty)$
   - B $(0, \infty)$
   - C $[0, \infty)$
   - D $(2, \infty)$
   - E $[2, \infty)$ (CORRECT ANSWER)

8. The phase amplitude ($|a|$), shift ($b$), and period ($p$) of $f(x) = 10 \cos \left( \frac{1}{2}x \right)$ are
   - A $|a| = 10, \ b = \frac{1}{2}, \ p = 4\pi$
   - B $|a| = -10, \ b = 2, \ p = 2\pi$
   - C $|a| = 10, \ b = 0, \ p = \frac{\pi}{4}$
   - D $|a| = 10, \ b = 0, \ p = \frac{\pi}{2}$
   - E None of the above (CORRECT ANSWER)

9. A polynomial with degree 3 and zeros $-1, 1, 3$ is
   - A $x^3 - 3x^2 - x + 3$ (CORRECT ANSWER)
   - B $x^3 + 3x^2 - x - 3$
   - C $x^3 + x^2 - 5x + 3$
   - D $x^4 - 6x^2 + 8x - 3$
   - E None of the above
FR (60 points). Problems 1-9 are free response questions. You are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. (5 points) Factor $x^4 - x^3 - 2x^2 + 2x$.

2. (10 points) Solve the following inequalities:
   
   (a) $2|2x - 7| < 14$

   (b) $(x - 1)(x + 2) \leq -2$
3. (15 points) Solve the following equations:

(a) \(2x - 1 = -\sqrt{2 - x}\)

(b) \(\log_9 (x - 5) + \log_9 (x + 3) = 1\)

(c) \(1 + \sin x = 2 \cos^2 x\)
4. (5 points) Combine the expression $\frac{1}{3} \log(x + 2) + \frac{1}{7} \log(x^2 + 1) - 5 \log(x^4 + 1) + \log x$.

5. (5 points) Show that the equation $x^2 + y^2 + 2x - 6y + 9 = 0$ represents a circle, and find the radius and center of the circle.
6. (5 points) Plot \( f(x) = \frac{x - 2}{x - 1} \) using transformations.

7. (5 points) Let \( P(1, 2) \) and \( Q(2, -1) \) be two points on the coordinate plane. Find the slope of a line that is perpendicular to the line through \( P \) and \( Q \).
8. (5 points) Find the inverse function of \( f(x) = -\sqrt{x - 1} \).

9. (5 points) A manufacturer of soft drinks advertises their orange soda as “naturally flavored,” although it contains only 5% orange juice. A new federal regulation stipulates that to be called “natural” a drink must contain at least 10% fruit juice. How much pure orange juice must this manufacturer add to 900 gal of orange soda to conform to the new regulation?