Final Exam

MATH-AD.101: Mathematical Functions

December 17, 2011

PLEASE READ THE FOLLOWING INFORMATION.

• This is a **110-minute** exam. Calculators, books, notes, and other aids are not allowed.

• You may use the backs of the pages or the extra pages for scratch work. **Do not unstaple or remove pages as they can be lost in the grading process.**

• Please do not put your name on any page besides the first page.
MC (42 points). This part consists of 21 multiple choice problems. Nothing more than the answer is required; consequently no partial credit will be awarded.

1. Simplify \( \left( \frac{x^{-2}y^{-1}}{2x^{5/2}y^{3/2}} \right)^{-2} \).
   - (A) \(-2xy^5\)
   - (B) \(2x^9y^4\)
   - (C) \(4x^2y^3\)
   - (D) \(4x^9y^5\)
   - (E) \(8xy^2\)

2. Simplify \( \frac{y^2 - x^2}{x - \frac{y}{1} - \frac{1}{y} - \frac{1}{x}} \).
   - (A) \(-x^2 - xy + y^2\)
   - (B) \(-x^2 + xy - y^2\)
   - (C) \(x^2 - xy - y^2\)
   - (D) \(x^2 + xy - y^2\)
   - (E) \(-x^2 - xy - y^2\)

3. Factor \( x^3 + 2x^2 - x - 2 \).
   - (A) \((x - 1)(x - 1)(x - 2)\)
   - (B) \((x - 1)(x + 1)(x - 2)\)
   - (C) \((x - 1)^2(x + 2)\)
   - (D) \((x - 1)(x + 1)(x + 2)\)
   - (E) \((x + 1)^2(x + 2)\)
4. Find all real solutions of \( \frac{1}{3}x - \frac{1}{2} = 1 + \frac{1}{2}x \).

A. \(-5\)
B. \(-6\)
C. \(-7\)
D. \(-8\)
E. \(-9\)

5. Find all real solutions of \( x^4 + x^2 - 6 = 0 \).

A. \(\sqrt{2}, \sqrt{3}\)
B. \(-\sqrt{2}, \sqrt{2}\)
C. \(-\sqrt{2}, \sqrt{2}, -\sqrt{3}, \sqrt{3}\)
D. \(-\sqrt{3}, \sqrt{3}\)
E. \(-\sqrt{2}, -\sqrt{3}\)

6. Solve the inequality \( \frac{x + 1}{3x + 2} \geq 1 \).

A. \(\left( \frac{2}{3}, \frac{1}{2} \right]\)
B. \(\left( -\frac{2}{3}, \frac{1}{2} \right]\)
C. \(\left( \frac{2}{3}, -\frac{1}{2} \right]\)
D. \(\left( -\frac{2}{3}, -\frac{1}{2} \right]\)
E. \(\left( -\frac{1}{2}, -\frac{2}{3} \right]\)
7. Solve the inequality $|2x - 1| < 5$.

- **A** ($-2, 3$)
- **B** ($2, -3$)
- **C** ($-2, -3$)
- **D** ($-2, -3$)
- **E** $[-2, -3]$

8. Find the center and radius of the circle $x^2 - 4x + y^2 + 2y + 2 = 0$.

- **A** Center ($2, -1$), radius $\sqrt{3}$
- **B** Center ($-2, 1$), radius $\sqrt{3}$
- **C** Center ($1, 2$), radius $3$
- **D** Center ($1, -2$), radius $3$
- **E** Center ($-1, -2$), radius $\sqrt{3}$

9. Find an equation for the line that passes through the point $(3, -6)$ and is parallel to the line $3x + y - 10 = 0$.

- **A** $y = -3x + 2$
- **B** $y = -3x - 3$
- **C** $y = 3x + 3$
- **D** $y = -3x + 3$
- **E** $y = 3x - 3$
10. Find the domain of \( \frac{x}{\sqrt{1-x}} \).

(A) 1
(B) \((-\infty, 0) \cup (0, \infty)\)
(C) All real numbers
(D) \((1, \infty)\)
(E) \((-\infty, 1)\)

11. How is the graph of \( y = f(x - 2) + 5 \) obtained from the graph of \( f \)?

(A) The graph shifts right 2 units, then shifts upward 5 units.
(B) The graph shifts right 5 units, then shifts upward 2 units.
(C) The graph shifts left 5 units, then shifts upward 2 units.
(D) The graph shifts right 5 units, then shifts downward 2 units.
(E) The graph shifts left 5 units, then shifts downward 2 units.

12. If \( f(x) = -\sqrt{x} \), find the inverse function \( f^{-1} \).

(A) \( x^2 \)
(B) \( \frac{1}{\sqrt{x}} \)
(C) \( \sqrt{x} \)
(D) \( x^2, x \leq 0 \)
(E) \( x^2, x \geq 0 \)
13. Evaluate \( \log_2 56 - \log_2 7 \).

- A) -1
- B) 0
- C) 1
- D) 2
- E) 3

14. Combine into a single logarithm \( \ln x - 2 \ln(x^2 + 1) + \frac{1}{2} \ln(x^4 + 1) \).

- A) \( \ln \frac{x(x^2 + 1)^2}{\sqrt{x^4 + 1}} \)
- B) \( \ln \frac{(x^2 + 1)^2}{x\sqrt{x^4 + 1}} \)
- C) \( \ln \frac{\sqrt{x^4 + 1}}{x(x^2 + 1)^2} \)
- D) \( \ln \frac{x\sqrt{x^4 + 1}}{(x^2 + 1)^2} \)
- E) \( \ln \frac{\sqrt{x^4 + 1}}{(x^2 + 1)^2} \)

15. Find the exact value \( \cot \left( -\frac{5\pi}{3} \right) \).

- A) \( \frac{1}{\sqrt{3}} \)
- B) \( -\frac{1}{\sqrt{3}} \)
- C) \( \sqrt{3} \)
- D) \( -\sqrt{3} \)
- E) 1
16. Find the exact value of $\sin 10^\circ \cos 20^\circ + \cos 10^\circ \sin 20^\circ$.

A $\frac{\sqrt{2}}{2}$
B $\frac{\sqrt{3}}{2}$
C 1
D 0
E $\frac{1}{2}$

17. Find the exact value of $\cos \left( \tan^{-1} \frac{3}{4} \right)$.

A $\frac{4}{5}$
B $\frac{5}{4}$
C $\frac{3}{2}$
D $\frac{2}{3}$
E $\frac{2}{5}$

18. Simplify $\tan \theta \sin \theta + \cos \theta$.

A $\cos \theta$
B $\sec \theta$
C $\tan \theta$
D $\sin \theta$
E $\cot \theta$
19. Convert the equation \( r = 2 \cos \theta \) to rectangular coordinates.
   \[ \text{A} \quad (x - 1)^2 + (y - 1)^2 = 1 \]
   \[ \text{B} \quad x^2 + y^2 = 1 \]
   \[ \text{C} \quad x^2 + (y - 1)^2 = 1 \]
   \[ \text{D} \quad (x - 1)^2 + (y + 1)^2 = 1 \]
   \[ \text{E} \quad (x - 1)^2 + y^2 = 1 \]

20. Let \( \mathbf{u} = \langle 1, 3 \rangle \) and \( \mathbf{v} = \langle -6, 2 \rangle \). Find \( \mathbf{u} \cdot \mathbf{v} \).
   \[ \text{A} \quad 0 \]
   \[ \text{B} \quad 1 \]
   \[ \text{C} \quad 2 \]
   \[ \text{D} \quad 3 \]
   \[ \text{E} \quad 4 \]

21. Find all solutions of the system \[
\begin{align*}
\frac{1}{2} + \frac{1}{2}y &= 1 \\
x + y &= \frac{1}{2}
\end{align*}
\]
   \[ \text{A} \quad x = \frac{1}{2}, \; y = \frac{3}{2} \]
   \[ \text{B} \quad x = \frac{3}{2}, \; y = -1 \]
   \[ \text{C} \quad x = \frac{3}{2}, \; y = \frac{1}{2} \]
   \[ \text{D} \quad x = \frac{3}{2}, \; y = 1 \]
   \[ \text{E} \quad x = 1, \; y = \frac{1}{2} \]
FR (40 points). Problems FR1–FR4 are free response questions. Put your answers in the boxes (where provided) and your work/explanations in the space below the problem.

- Read and follow the instructions of every problem.
- Show all of your work for purposes of partial credit. **Full credit may not be given for an answer alone.**
- Justify your answers. **Full sentences are not necessary**, but English words help. When in doubt, do as much as you think is necessary to demonstrate that you understand the problem, keeping in mind that your grader will be necessarily skeptical.

**FR1** (10 points). Mary drove from Amity to Belleville at a speed of 50 mi/h. On the way back, she drove at 60 mi/h. The total trip took \( \frac{4\frac{2}{5}}{} \) h of driving time. Find the distance between these two cities.

\[
\text{Distance} = \boxed{\quad}\]
FR2 (10 points). Use transformations to plot the graph of $f(x) = 1 + \frac{1}{2} \sin(2x)$.

Graph of $f(x) = 1 + \frac{1}{2} \sin(2x)$ is
**FR3** (10 points). Let \( f(x) = \frac{x + 2}{x - 3} \).

(a) Find the vertical and horizontal asymptote(s).

Vertical asymptote(s) =

Horizontal asymptote(s) =

(b) Sketch the graph of \( f \).
FR4 (10 points). If 1800 ft of fencing is available to build five adjacent pens, as shown in the diagram below, express the total area of the pens as a function of $x$. What value of $x$ will maximize the total area?

![Diagram of five adjacent pens with fencing]

$x = \underline{\phantom{0}}$
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