Section 7.5 More Trigonometric Equations

EXAMPLE: Solve the equation $1 + \sin x = 2\cos^2 x$.

Solution: We have

$$1 + \sin x = 2\cos^2 x$$

$$1 + \sin x = 2(1 - \sin^2 x)$$

$$1 + \sin x = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0 \qquad \text{or} \qquad \sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \qquad\qquad \sin x = -1$$

$$x = \frac{\pi}{6}, \ \frac{5\pi}{6} \qquad\qquad x = \frac{3\pi}{2}$$

Because sine has period 2π, we get all the solutions of the equation by adding any integer multiples of 2π to these solutions. Thus the solutions are

$$x = \frac{\pi}{6} + 2k\pi \qquad x = \frac{5\pi}{6} + 2k\pi \qquad x = \frac{3\pi}{2} + 2k\pi$$

where k is any integer.

EXAMPLE: Solve the equation $\sin 2x - \cos x = 0$.

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Solution: We have

$$\sin 2x - \cos x = 0$$

$$2\sin x\cos x - \cos x = 0$$

$$\cos x(2\sin x - 1) = 0$$

$$\cos x = 0 \qquad \text{or} \qquad 2\sin x - 1 = 0$$

$$\cos x = 0 \qquad\qquad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad\qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Both sine and cosine have period 2π, so we get all the solutions of the equation by adding any integer multiples of 2π to these solutions. Thus the solutions are

$$x = \frac{\pi}{2} + 2k\pi \qquad x = \frac{3\pi}{2} + 2k\pi \qquad x = \frac{\pi}{6} + 2k\pi \qquad x = \frac{5\pi}{6} + 2k\pi$$

where k is any integer.

EXAMPLE: Solve the equation $\cos x + 1 = \sin x$ in the interval $[0, 2\pi)$.

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Solution: We have

$$\cos x + 1 = \sin x$$

$$(\cos x + 1)^2 = \sin^2 x$$

$$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$$

$$2\cos^2 x + 2\cos x = 0$$

$$2\cos x(\cos x + 1) = 0$$

$$2\cos x = 0 \qquad \text{or} \qquad \cos x + 1 = 0$$

$$\cos x = 0 \qquad\qquad \cos x = -1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad\qquad x = \pi$$

Because we squared both sides, we need to check for extraneous solutions. Plugging in the numbers $\pi/2, 3\pi/2$, and $x = \pi$ into the original equation, we see that the solutions of the given equation are $\pi/2$ and π only.

EXAMPLE: Solve the following equations

(a) $\tan 3x + 1 = \sec 3x$ (b) $3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0$

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(a) $\tan 3x + 1 = \sec 3x$ (b) $3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0$

Solution:

(a) We have

$$\tan 3x + 1 = \sec 3x$$

$$(\tan 3x + 1)^2 = \sec^2 3x$$

$$\tan^2 3x + 2\tan 3x + 1 = \sec^2 3x$$

$$\sec^2 3x + 2\tan 3x = \sec^2 3x$$

$$2\tan 3x = 0$$

$$\tan 3x = 0$$

Therefore

$$3x = k\pi \quad \Longrightarrow \quad x = \frac{k\pi}{3}$$

(b) We have

$$3\tan^3 x - 3\tan^2 x - \tan x + 1 = 0$$

$$3\tan^2 x(\tan x - 1) - (\tan x - 1) = 0$$

$$(\tan x - 1)(3\tan^2 x - 1) = 0$$

$$\tan x - 1 = 0 \qquad \text{or} \qquad 3\tan^2 x - 1 = 0$$

$$\tan x = 1 \qquad\qquad \tan x = \pm\frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{4} \qquad\qquad x = \pm\frac{\pi}{6}$$

The tangent function has period π, so we get all the solutions of the equation by adding any integer multiples of π to these solutions. Thus the solutions are

$$x = \frac{\pi}{4} + k\pi \qquad x = \frac{\pi}{6} + k\pi \qquad x = -\frac{\pi}{6} + k\pi$$

where k is any integer.