Section 7.4 Trigonometric Equations

EXAMPLE: Solve the equation $\cos x = -\dfrac{\sqrt{2}}{2}$, and list eight specific solutions.

Solution:

Find solutions in one period. Because cosine has period 2π, we first find the solutions in any interval of length 2π. The solutions are

$$x = \frac{3\pi}{4} \qquad\qquad x = \frac{5\pi}{4}$$

Find all solutions. Because the cosine function repeats its value every 2π units, we get all solutions of the equation by adding integer multiples of 2π to these solutions:

$$x = \frac{3\pi}{4} + 2k\pi \qquad\qquad x = \frac{5\pi}{4} + 2k\pi$$

where k is any integer. For $k = -1, 0, 1, 2$ we get the following specific solutions:

$$x = -\frac{5\pi}{4}, \quad -\frac{3\pi}{4}, \quad \frac{3\pi}{4}, \quad \frac{5\pi}{4}, \quad \frac{11\pi}{4}, \quad \frac{13\pi}{4}, \quad \frac{19\pi}{4}, \quad \frac{21\pi}{4}$$

EXAMPLE: Solve the equation $2\sin x - 1 = 0$.

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Solution:

Find solutions in one period. We have

$$2\sin x - 1 = 0$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

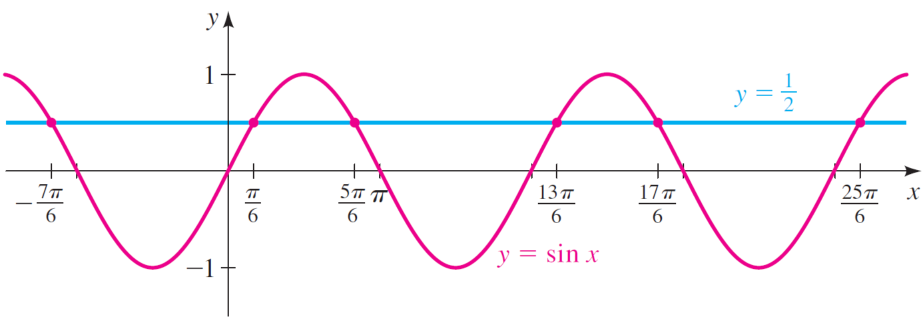
Because sine has period 2π, we first find the solutions in any interval of length 2π. The solutions are

$$x = \frac{\pi}{6} \qquad\qquad x = \frac{5\pi}{6}$$

Find all solutions. Because the sine function repeats its value every 2π units, we get all solutions of the equation by adding integer multiples of 2π to these solutions:

$$x = \frac{\pi}{6} + 2k\pi \qquad\qquad x = \frac{5\pi}{6} + 2k\pi$$

where k is any integer.



EXAMPLE: Solve the equation $\tan x = 2$.

Solution:

Find solutions in one period. We have

$$\tan x = 2$$

$$x = \tan^{-1} 2 \approx 1.12$$

By definition of $\tan^{-1}$, the solution that we obtained is the only solution in the interval $(-\pi/2, \pi/2)$.

Find all solutions. Because the tangent has period π, we get all solutions of the equation by adding integer multiples of π:

$$x \approx 1.12 + k\pi$$

where k is any integer.

EXAMPLE: Find all solutions of the equation $\tan^2 x - 3 = 0$.

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Solution: We have

$$\tan^2 x - 3 = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm\sqrt{3}$$

Because tangent has period π, we first find the solutions in any interval of length π. In the interval $(-\pi/2, \pi/2)$ the solutions are $x = \pi/3$ and $x = -\pi/3$. To get all solutions, we add integer multiples of π to these solutions:

$$x = \frac{\pi}{3} + k\pi \qquad\qquad x = -\frac{\pi}{3} + k\pi$$

where k is any integer.

EXAMPLE: Solve the equation $2\cos^2 x - 7\cos x + 3 = 0$.

Solution: We have

$$2\cos^2 x - 7\cos x + 3 = 0$$

$$(2\cos x - 1)(\cos x - 3) = 0$$

$$2\cos x - 1 = 0 \qquad \text{or} \qquad \cos x - 3 = 0$$

$$\cos x = \frac{1}{2} \qquad\qquad \cos x = 3$$

Because cosine has period 2π, we first find the solutions in the interval $[0, 2\pi)$. For the first equation the solutions are $x = \pi/3$ and $x = 5\pi/3$. The second equation has no solution because $\cos x$ is never greater that 1. Thus the solutions are

$$x = \frac{\pi}{3} + 2k\pi \qquad\qquad x = \frac{5\pi}{3} + 2k\pi$$

where k is any integer.

EXAMPLE: Solve the equation $5\sin x\cos x + 4\cos x = 0$.

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Solution: We have

$$5\sin x\cos x + 4\cos x = 0$$

$$\cos x(5\sin x + 4) = 0$$

$$\cos x = 0 \qquad \text{or} \qquad 5\sin x + 4 = 0$$

$$\cos x = 0 \qquad\qquad \sin x = -0.8$$

Because sine and cosine have period 2π, we first find the solutions in the interval of length 2π. For the first equation the solutions in the interval $[0, 2\pi)$ are $x = \pi/2$ and $x = 3\pi/2$. The solution of the second equation is

$$x = \sin^{-1}(-0.8) \approx -0.93$$

So the solutions in an interval of length 2π are $x \approx -0.93$ and $x \approx \pi + 0.93 \approx 4.07$. We get all the solutions of the equation by adding integer multiplers to these solutions.

$$x = \frac{\pi}{2} + 2k\pi \qquad x = \frac{3\pi}{2} + 2k\pi \qquad x \approx -0.93 + 2k\pi \qquad x \approx 4.07 + 2k\pi$$

where k is any integer.