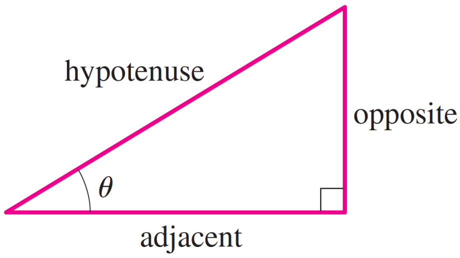
Section 6.2 Trigonometry of Right Triangles

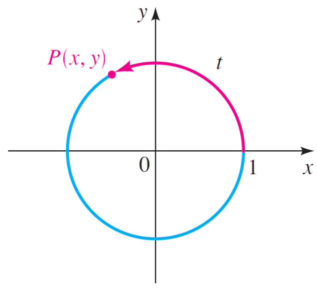
Trigonometric Ratios

The Trigonometric Ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$





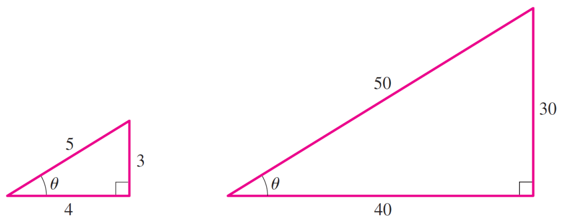
Definition of the Trigonometric Functions

Let t be any real number and let $P(x, y)$ be the terminal point on the unit circle determined by t. We define

$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x} \quad (x \neq 0)$$

$$\csc t = \frac{1}{y} \quad (y \neq 0) \qquad \sec t = \frac{1}{x} \quad (x \neq 0) \qquad \cot t = \frac{x}{y} \quad (y \neq 0)$$

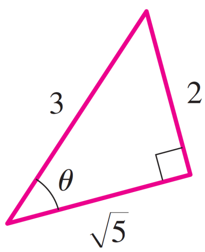
Since any two right triangles with angle θ are similar, the trigonometric ratios are the same, regardless of the size of the triangle; the trigonometric ratios depend only on the angle θ.



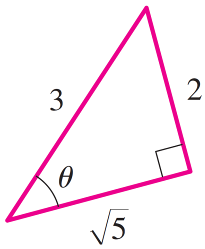
$\sin\theta = \frac{3}{5}$ $\qquad$ $\sin\theta = \frac{30}{50} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$ $\qquad$ $\cos\theta = \frac{40}{50} = \frac{4}{5}$

EXAMPLE: Find the six trigonometric ratios of the angle θ in the Figure on the right.



EXAMPLE: Find the six trigonometric ratios of the angle θ in the Figure on the right.

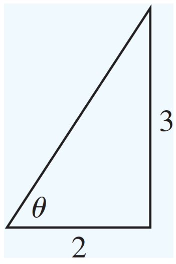


Solution: We have

$$\sin\theta = \frac{2}{3} \qquad \cos\theta = \frac{\sqrt{5}}{3} \qquad \tan\theta = \frac{2}{\sqrt{5}}$$

$$\csc\theta = \frac{3}{2} \qquad \sec\theta = \frac{3}{\sqrt{5}} \qquad \cot\theta = \frac{\sqrt{5}}{2}$$

EXAMPLE: Find the six trigonometric ratios of the angle θ in the Figure on the right.



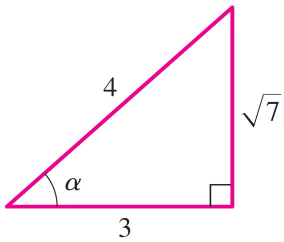
Solution: We have

$$\sin\theta = \frac{3}{\sqrt{13}} \qquad \cos\theta = \frac{2}{\sqrt{13}} \qquad \tan\theta = \frac{3}{2}$$

$$\csc\theta = \frac{\sqrt{13}}{3} \qquad \sec\theta = \frac{\sqrt{13}}{2} \qquad \cot\theta = \frac{2}{3}$$

EXAMPLE: If $\cos\alpha = \dfrac{3}{4}$, sketch a right triangle with acute angle α, and find the other five trigonometric ratios of α.

Solution: We have



$$\sin\theta = \frac{\sqrt{7}}{4} \qquad \cos\theta = \frac{3}{4} \qquad \tan\theta = \frac{\sqrt{7}}{3}$$

$$\csc\theta = \frac{4}{\sqrt{7}} \qquad \sec\theta = \frac{4}{3} \qquad \cot\theta = \frac{3}{\sqrt{7}}$$

EXAMPLE: Consider a right triangle with α as one of its acute angles. If $\tan\alpha = \dfrac{7}{8}$, find the other five trigonometric ratios of α.

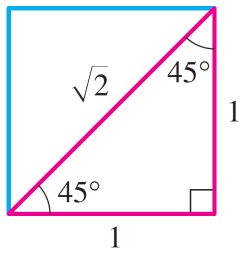
Solution: We have

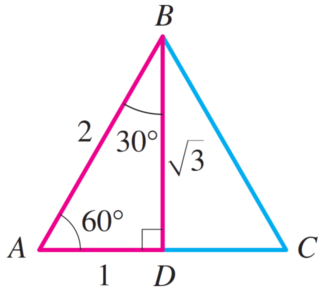
$$\sin\theta = \frac{7}{\sqrt{113}} \qquad \cos\theta = \frac{8}{\sqrt{113}} \qquad \tan\theta = \frac{7}{8}$$

$$\csc\theta = \frac{\sqrt{113}}{7} \qquad \sec\theta = \frac{\sqrt{113}}{8} \qquad \cot\theta = \frac{8}{7}$$

Special Triangles

Certain right triangles have ratios that can be calculated easily from the Pythagorean Theorem. The first triangle is obtained by drawing a diagonal in a square of side 1 (see the first Figure below). By the Pythagorean Theorem this diagonal has length $\sqrt{2}$. Similarly, by the Pythagorean Theorem the length of DB (see the second Figure below) is $\sqrt{3}$.





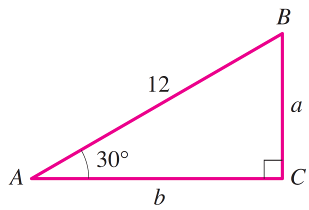
We can now use the special triangles in the Figures above to calculate the trigonometric ratios for angles with measures $30°, 45°$, and $60°$ (or $\pi/6, \pi/4$, and $\pi/3$).

Table 1 Values of the trigonometric ratios for special angles

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Applications of Trigonometry of Right Triangles

EXAMPLE: Solve triangle ABC, shown on the right.



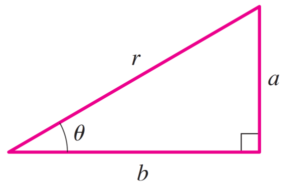
Solution: It's clear that $\angle B = 60°$. To find a, we look for an equation that relates a to the lengths and angles we already know. In this case, we have $\sin 30° = a/12$, so

$$a = 12\sin 30° = 12\left(\frac{1}{2}\right) = 6$$

Similarly, $\cos 30° = b/12$, so

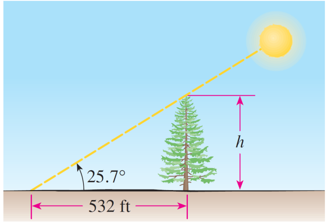
$$b = 12\cos 30° = 12\left(\frac{\sqrt{3}}{2}\right) = 6\sqrt{3}$$

It's very useful to know that, using the information given in the Figure on the right, the lengths of the legs of a right triangle are



$$a = r\sin\theta \qquad \text{and} \qquad b = r\cos\theta$$

EXAMPLE: A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is 25.7°.



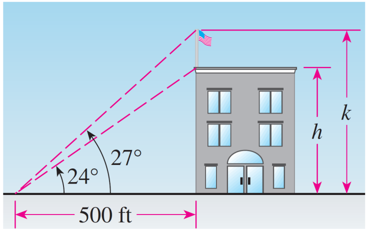
Solution: Let the height of the tree be h. From the Figure on the right we see that

$$\frac{h}{532} = \tan 25.7^\circ$$

$$h = 532\tan 25.7^\circ \approx 532(0.48127) \approx 256$$

The height of the tree is about 256 ft.

EXAMPLE: From a point on the ground 500 ft from the base of a building, an observer finds that the angle of elevation to the top of the building is 24° and that the angle of elevation to the top of a flagpole atop the building is 27°. Find the height of the building and the length of the flagpole.



Solution: The Figure on the right illustrates the situation. The height of the building is found in the same way that we found the height of the tree in the previous Example.

$$\frac{h}{500} = \tan 24^\circ$$

$$h = 500\tan 24^\circ \approx 500(0.4452) \approx 223$$

The height of the tree is about 223 ft.

To find the length of the flagpole, let's first find the height from the ground to the top of the pole:

$$\frac{k}{500} = \tan 27^\circ$$

$$k = 500\tan 27^\circ \approx 500(0.5095) \approx 255$$

To find the length of the flagpole, we subtract h from k. So the length of the pole is approximately $255 - 223 = 32$ ft.