Section 2.3 Getting Information from the Graph of a Function

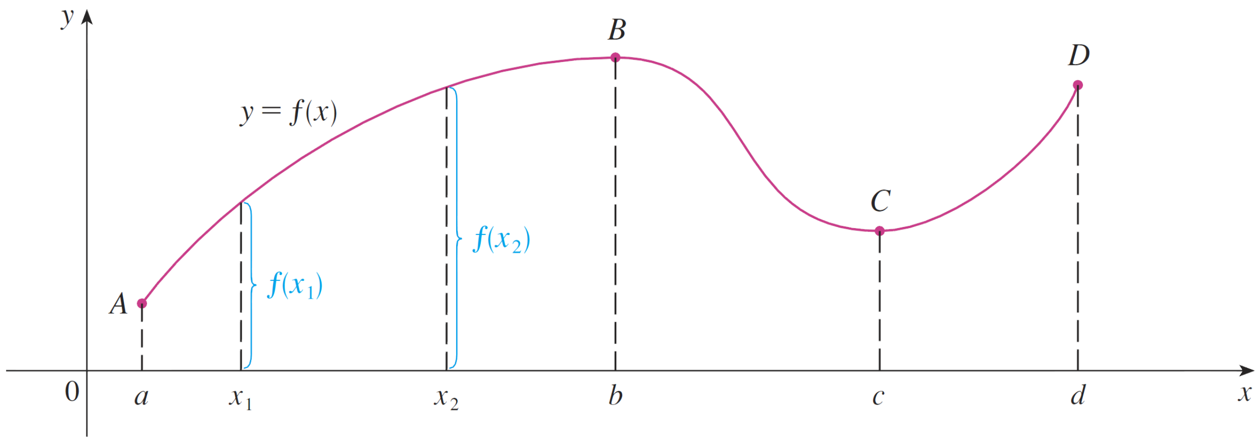
Increasing and Decreasing Functions

DEFINITION: A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on an I if

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

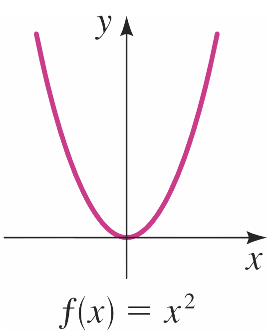


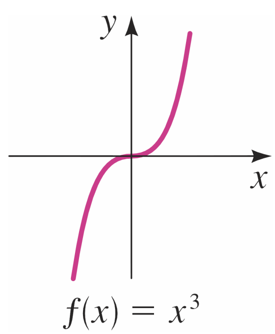
EXAMPLES:

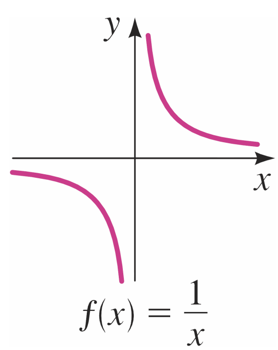
1. The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

2. The function $f(x) = x^3$ is increasing everywhere, that is on $(-\infty, \infty)$.

3. The function $f(x) = \dfrac{1}{x}$ is decreasing on $(-\infty, 0)$ and on $(0, \infty)$.

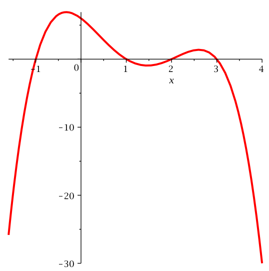


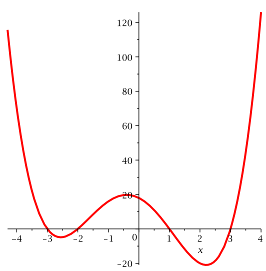


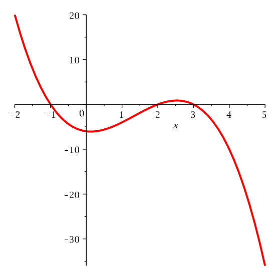


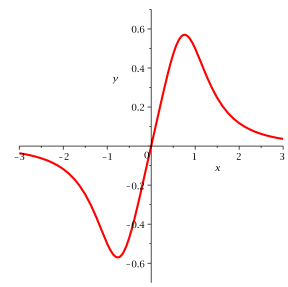
Local Maximum and Minimum Values of a Function

DEFINITION: A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. [This means that $f(c) \geq f(x)$ for all x in some open interval containing c.] Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.









EXAMPLES:

1. The function

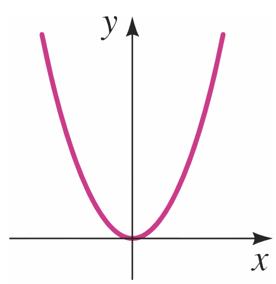
$$f(x) = x^2$$

has a local minimum at $x = 0$ and has no local maximum.

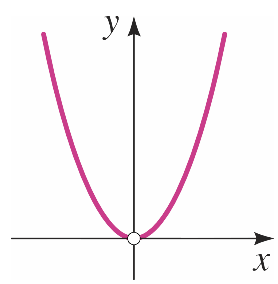
2. The function

$$f(x) = x^2, \ x \in (-\infty, 0) \cup (0, \infty)$$

has no local minimum or maximum.



$f(x) = x^2$

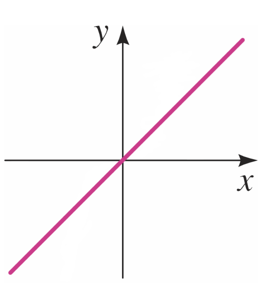


$f(x) = x^2, \ x \in (-\infty, 0) \cup (0, \infty)$

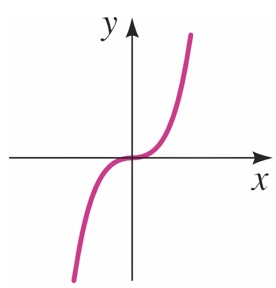
3. The functions

$$f(x) = x, \quad x^3, \quad x^5$$

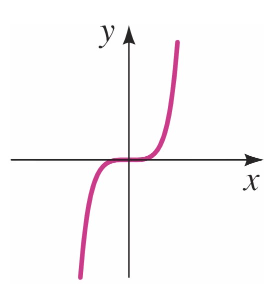
have no local minima or maxima.



$f(x) = x$



$f(x) = x^3$



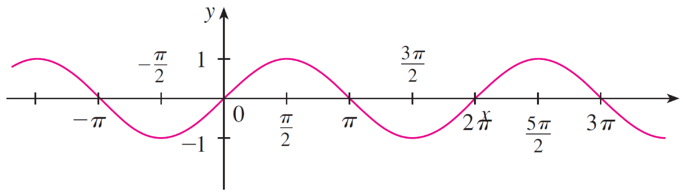
$f(x) = x^5$

DEFINITION: A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. [This means that $f(c) \geq f(x)$ for all x in some open interval containing c.] Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

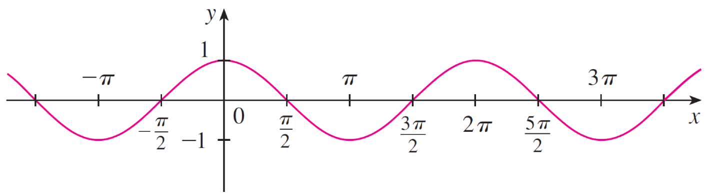
4. The functions

$$f(x) = \sin x, \quad \cos x, \quad \sec x, \quad \csc x$$

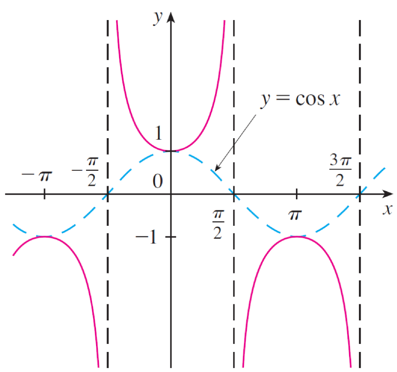
have infinitely many local minima and maxima.



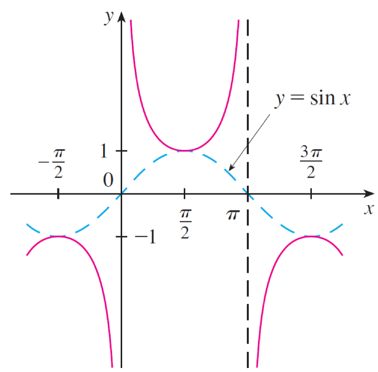
$y = \sin x$



$y = \cos x$



$y = \sec x$

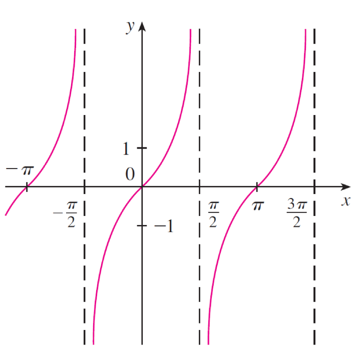


$y = \csc x$

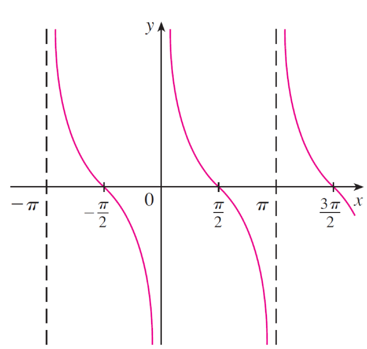
5. The functions

$$f(x) = \tan x, \quad \cot x$$

have no local minima or maxima.



$y = \tan x$



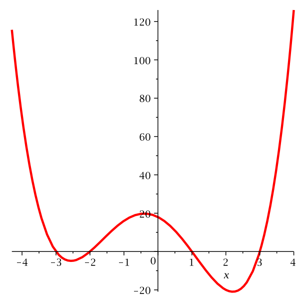
$y = \cot x$

DEFINITION: A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. [This means that $f(c) \geq f(x)$ for all x in some open interval containing c.] Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

6. The function

$$f(x) = x^4 + x^3 - 11\,x^2 - 9\,x + 18 = (x-3)(x-1)(x+2)(x+3)$$

has two local minima at $x \approx -2.6$ and $x \approx 2.2$ and a local maximum at $x \approx -0.4$.



7. The function

$$f(x) = 1$$

has a local minimum and maximum at any point on the number line.