

Section 2.2 Graphs of Functions

DEFINITION: A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B . It's **graph** is the set of ordered pairs

$$\{(x, f(x)) \mid x \in A\}$$

EXAMPLE:

Sketch the graphs of the following functions.

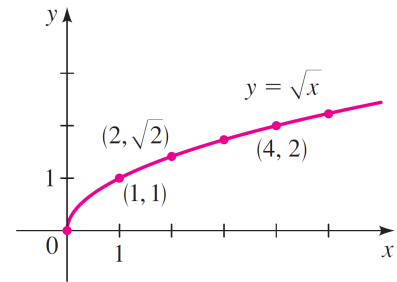
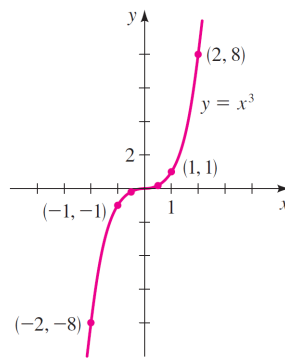
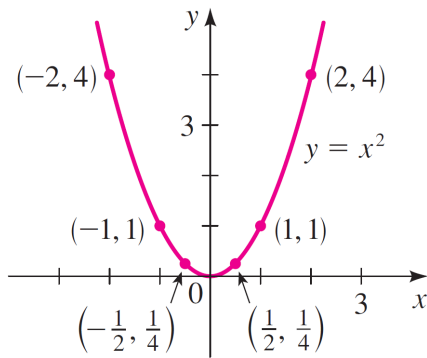
(a) $f(x) = x^2$ (b) $g(x) = x^3$ (c) $h(x) = \sqrt{x}$

Solution: We first make a table of values. Then we plot the points given by the table and join them by a smooth curve to obtain the graph. The graphs are sketched in the Figures below.

x	$f(x) = x^2$
0	0
$\pm \frac{1}{2}$	$\frac{1}{4}$
± 1	1
± 2	4
± 3	9

x	$g(x) = x^3$
0	0
$\frac{1}{2}$	$\frac{1}{8}$
1	1
2	8
$-\frac{1}{2}$	$-\frac{1}{8}$
-1	-1
-2	-8

x	$h(x) = \sqrt{x}$
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$
4	2
5	$\sqrt{5}$



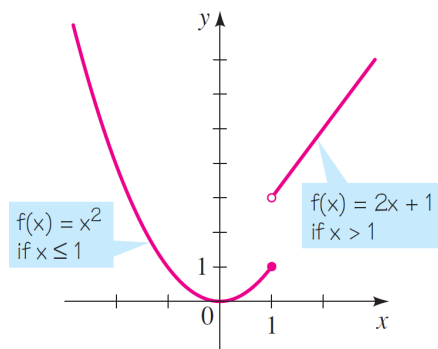
EXAMPLE: Sketch the graph of the function.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$$

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$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$$

Solution: If $x \leq 1$, then $f(x) = x^2$, so the part of the graph to the left of $x = 1$ coincides with the graph of $y = x^2$. If $x > 1$, then $f(x) = 2x + 1$, so the part of the graph to the right of $x = 1$ coincides with the line $y = 2x + 1$.

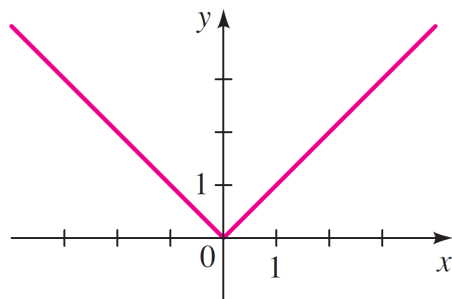


EXAMPLE: Sketch the graph of the function $f(x) = |x|$.

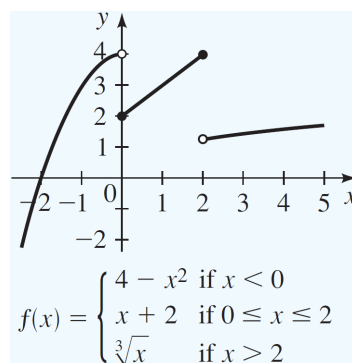
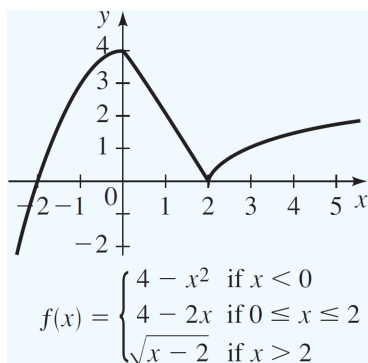
Solution: Recall that

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Using the same method as in the previous example, we note that the graph of f coincides with the line $y = x$ to the right of the y -axis and coincides with the line $y = -x$ to the left of the y -axis.

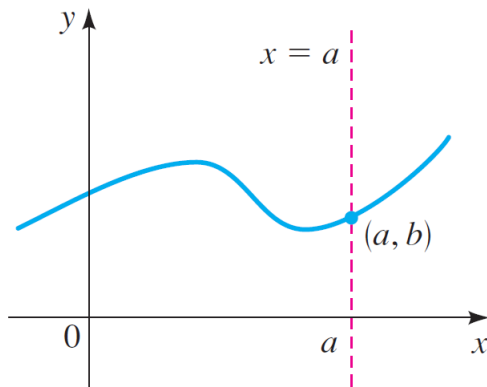


EXAMPLES:

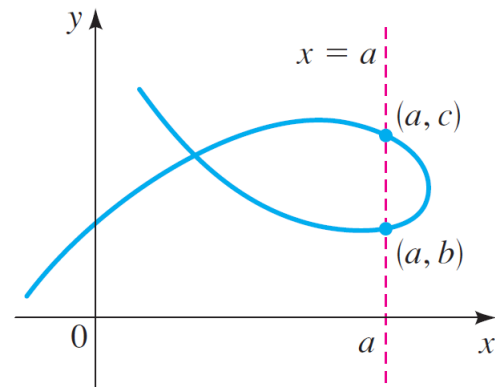


The Vertical Line Test

A curve in the coordinate plane is the graph of a function if and only if no vertical line intersects the curve more than once.

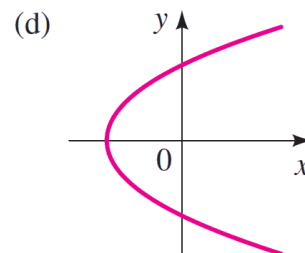
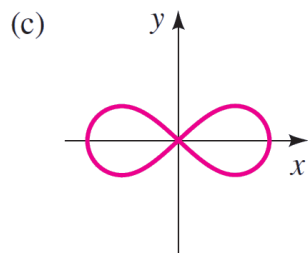
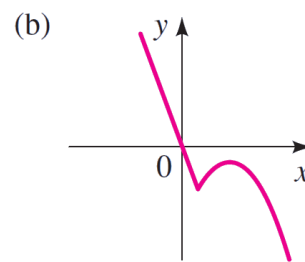
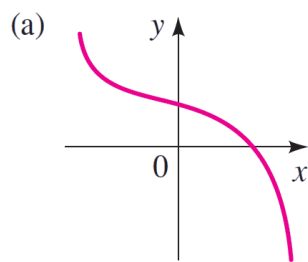


Graph of a function



Not a graph of a function

EXAMPLE: Which of the following are graphs of functions?

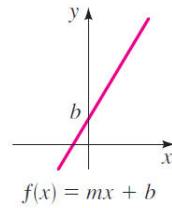
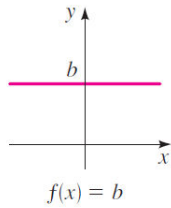


Solution: (a) and (b) are graphs of functions, (c) and (d) are not.

Some Functions and Their Graphs

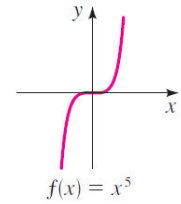
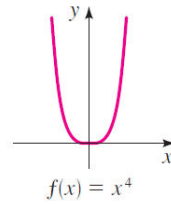
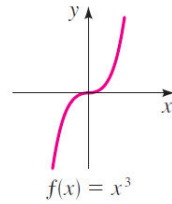
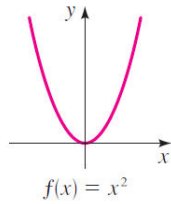
Linear functions

$$f(x) = mx + b$$



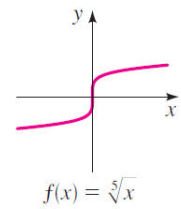
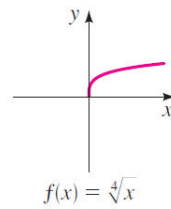
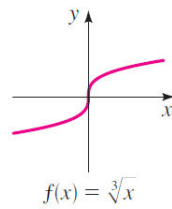
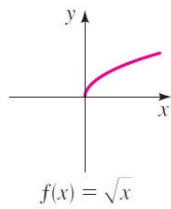
Power functions

$$f(x) = x^n$$



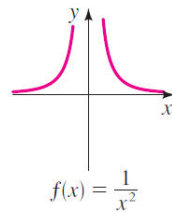
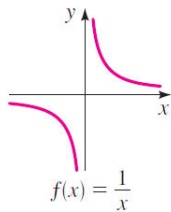
Root functions

$$f(x) = \sqrt[n]{x}$$



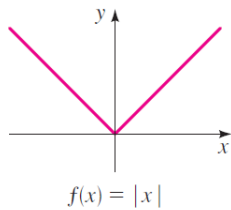
Reciprocal functions

$$f(x) = 1/x^n$$



Absolute value function

$$f(x) = |x|$$



Greatest integer function

$$f(x) = \llbracket x \rrbracket$$

