Section 10.2 Systems of Linear Equations in Several Variables

EXAMPLE: Find all solutions of the system

$$\begin{cases} x+y+z=1 \\ y+z=1 \\ 2z=4 \end{cases}$$

Solution 1(Substitution Method): We solve for z in the last equation.

$$2z=4 \iff z=2$$

Now we substitute for z in the second equation and solve for y:

$$y+z=1$$

$$y+2=1$$

$$y=-1$$

Finally, we substitute $y=-1$ and $z=2$ into the first equation $x+y+z=1$:

$$x+y+z=1$$

$$x+(-1)+2=1$$

$$x=0$$

Solution 2(Elimination Method): We have

$$\begin{cases} x+y+z=1 \\ y+z=1 \\ 2z=4 \end{cases} \iff \begin{cases} x+y+z=1 \\ y+z=1 \\ z=2 \end{cases} \iff \begin{cases} x=0 \\ y+z=1 \\ z=2 \end{cases} \iff \begin{cases} x=0 \\ y=-1 \\ z=2 \end{cases}$$

EXAMPLE: Find all solutions of the system

$$\begin{cases} x+y+z=6 \\ x+2y-z=2 \\ x+2y+3z=14 \end{cases}$$

Solution (Elimination Method): We have

$$\begin{cases} x+y+z=6 \\ x+2y-z=2 \\ x+2y+3z=14 \end{cases} \iff \begin{cases} x+y+z=6 \\ y-2z=-4 \\ 4z=12 \end{cases} \iff \begin{cases} x+y+z=6 \\ y-2z=-4 \\ z=3 \end{cases} \iff \begin{cases} x+y+z=6 \\ y-2(3)=-4 \\ z=3 \end{cases}$$

therefore

$$\begin{cases} x+y+z=6 \\ y=2 \\ z=3 \end{cases} \iff \begin{cases} x+2+3=6 \\ y=2 \\ z=3 \end{cases} \iff \begin{cases} x=1 \\ y=2 \\ z=3 \end{cases}$$

EXAMPLE: Find all solutions of the system

$$\begin{cases} 2x-y+3z=1 \\ x-2y+z=1 \\ 2x-3y-z=2 \end{cases}$$

EXAMPLE: Find all solutions of the system

$$\begin{cases} 2x - y + 3z = 1 \\ x - 2y + z = 1 \\ 2x - 3y - z = 2 \end{cases}$$

Solution (Elimination Method): We have

$$\begin{cases} 2x - y + 3z = 1 \\ x - 2y + z = 1 \\ 2x - 3y - z = 2 \end{cases} \iff \begin{cases} 2x - y + 3z = 1 \\ 2x - 4y + 2z = 2 \\ 2x - 3y - z = 2 \end{cases} \iff \begin{cases} 2x - y + 3z = 1 \\ -3y - z = 1 \\ -2y - 4z = 1 \end{cases} \iff \begin{cases} 2x - y + 3z = 1 \\ -12y - 4z = 4 \\ -2y - 4z = 1 \end{cases}$$

therefore

$$\begin{cases} 2x - y + 3z = 1 \\ -12y - 4z = 4 \\ 10y = -3 \end{cases} \iff \begin{cases} 2x - y + 3z = 1 \\ -12y - 4z = 4 \\ y = -\dfrac{3}{10} \end{cases} \iff \begin{cases} 2x - y + 3z = 1 \\ -12\left(-\dfrac{3}{10}\right) - 4z = 4 \\ y = -\dfrac{3}{10} \end{cases} \iff \begin{cases} 2x - y + 3z = 1 \\ z = -\dfrac{1}{10} \\ y = -\dfrac{3}{10} \end{cases}$$

so

$$\begin{cases} 2x - \left(-\dfrac{3}{10}\right) + 3\left(-\dfrac{1}{10}\right) = 1 \\ z = -\dfrac{1}{10} \\ y = -\dfrac{3}{10} \end{cases} \iff \begin{cases} x = \dfrac{1}{2} \\ z = -\dfrac{1}{10} \\ y = -\dfrac{3}{10} \end{cases}$$