

Section 10.1 Systems of Linear Equations in Two Variables

EXAMPLE: Find all solutions of the system

$$\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases}$$

Solution 1(Substitution Method): We solve for x in the second equation.

$$x + 4y = 7 \iff x = 7 - 4y$$

Now we substitute for x in the first equation and solve for y :

$$\begin{aligned} 2x - y &= 5 \\ 2(7 - 4y) - y &= 5 \\ 14 - 8y - y &= 5 \\ 14 - 9y &= 5 \\ -9y &= -9 \\ y &= 1 \end{aligned}$$

Finally, we back-substitute $y = 1$ into the equation $x = 7 - 4y$:

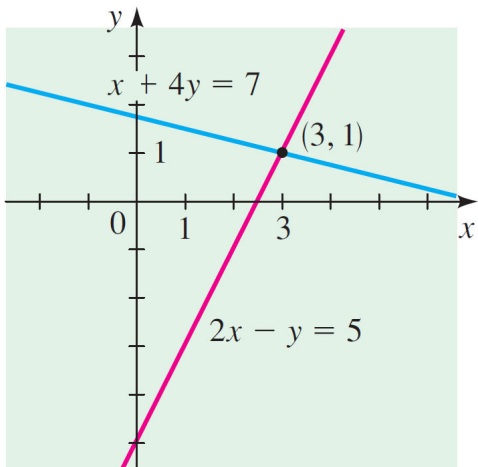
$$x = 7 - 4(1) = 3$$

Solution 2(Elimination Method): We have

$$\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases} \iff \begin{cases} 2x - y = 5 \\ 2x + 8y = 14 \end{cases} \iff \begin{cases} 2x - y = 5 \\ 9y = 9 \end{cases} \iff \begin{cases} 2x - y = 5 \\ y = 1 \end{cases}$$

Next we substitute $y = 1$ into the equation $2x - y = 5$:

$$2x - 1 = 5 \iff 2x = 6 \iff x = 3$$



EXAMPLE: Find all solutions of the system

$$\begin{cases} 2x + y = 1 \\ 3x + 4y = 14 \end{cases}$$

Solution 1(Substitution Method): We solve for y in the first equation.

$$2x + y = 1 \iff y = 1 - 2x$$

Now we substitute for y in the second equation and solve for x :

$$3x + 4y = 14$$

$$3x + 4(1 - 2x) = 14$$

$$3x + 4 - 8x = 14$$

$$-5x = 10$$

$$x = -2$$

Finally, we back-substitute $x = -2$ into the equation $y = 1 - 2x$:

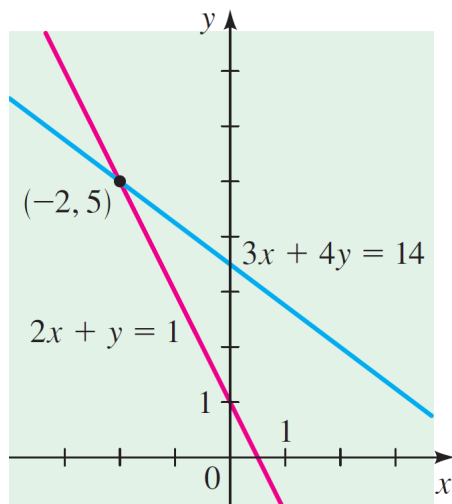
$$y = 1 - 2(-2) = 5$$

Solution 2(Elimination Method): We have

$$\begin{cases} 2x + y = 1 \\ 3x + 4y = 14 \end{cases} \iff \begin{cases} 8x + 4y = 4 \\ 3x + 4y = 14 \end{cases} \iff \begin{cases} 5x = -10 \\ 3x + 4y = 14 \end{cases} \iff \begin{cases} x = -2 \\ 3x + 4y = 14 \end{cases}$$

Next we substitute $x = -2$ into the equation $3x + 4y = 14$:

$$3(-2) + 4y = 14 \iff -6 + 4y = 14 \iff 4y = 20 \iff y = 5$$



EXAMPLE: Find all solutions of the system

$$\begin{cases} 2x + 3y = -4 \\ 5x - 7y = 1 \end{cases}$$

Solution 1(Substitution Method): We solve for x in the first equation.

$$2x + 3y = -4 \iff 2x = -4 - 3y \iff x = -\frac{4 + 3y}{2}$$

Now we substitute for x in the second equation and solve for y :

$$\begin{aligned} 5x - 7y &= 1 \\ -5\frac{4 + 3y}{2} - 7y &= 1 \\ -5(4 + 3y) - 14y &= 2 \\ -20 - 15y - 14y &= 2 \\ -29y &= 22 \\ y &= -\frac{22}{29} \end{aligned}$$

Finally, we back-substitute $y = -\frac{22}{29}$ into the equation $x = -\frac{4 + 3y}{2}$:

$$x = -\frac{4 + 3\left(-\frac{22}{29}\right)}{2} = -\frac{25}{29}$$

Solution 2(Elimination Method): On the one hand, we have

$$\begin{cases} 2x + 3y = -4 \\ 5x - 7y = 1 \end{cases} \iff \begin{cases} 10x + 15y = -20 \\ 10x - 14y = 2 \end{cases} \implies 29y = -22 \iff y = -\frac{22}{29}$$

On the other hand, we have

$$\begin{cases} 2x + 3y = -4 \\ 5x - 7y = 1 \end{cases} \iff \begin{cases} 14x + 21y = -28 \\ 15x - 21y = 3 \end{cases} \implies 29x = -25 \iff x = -\frac{25}{29}$$

EXAMPLE: Find all solutions of the system

$$\begin{cases} -\frac{3}{5}x + \frac{1}{2}y = \frac{7}{2} \\ \frac{1}{3}x + \frac{4}{5}y = \frac{3}{2} \end{cases}$$

EXAMPLE: Find all solutions of the system

$$\begin{cases} -\frac{3}{5}x + \frac{1}{2}y = \frac{7}{2} \\ \frac{1}{3}x + \frac{4}{5}y = \frac{3}{2} \end{cases}$$

Solution (Elimination Method): We have

$$\begin{cases} -\frac{3}{5}x + \frac{1}{2}y = \frac{7}{2} \\ \frac{1}{3}x + \frac{4}{5}y = \frac{3}{2} \end{cases} \iff \begin{cases} -6x + 5y = 35 \\ 10x + 24y = 45 \end{cases}$$

On the one hand, we have

$$\begin{cases} -6x + 5y = 35 \\ 10x + 24y = 45 \end{cases} \iff \begin{cases} -30x + 25y = 175 \\ 30x + 72y = 135 \end{cases} \implies 97y = 310 \iff y = \frac{310}{97}$$

On the other hand, we have

$$\begin{cases} -6x + 5y = 35 \\ 10x + 24y = 45 \end{cases} \iff \begin{cases} -144x + 120y = 840 \\ 50x + 120y = 225 \end{cases} \implies -194x = 615 \iff x = -\frac{615}{194}$$

EXAMPLE: Find all solutions of the system

$$\begin{cases} x + y = 1 \\ x + y = 2 \end{cases}$$

Answer: The system has no solution (**inconsistent**).

EXAMPLE: Find all solutions of the system

$$\begin{cases} 15x - 10y = 25 \\ -24x + 16y = -42 \end{cases}$$

Solution: We have

$$\begin{cases} 15x - 10y = 25 \\ -24x + 16y = -42 \end{cases} \iff \begin{cases} 3x - 2y = 5 \\ 3x - 2y = \frac{21}{4} \end{cases} \iff \begin{cases} 3x - 2y = 5 \\ 0 = \frac{1}{4} \end{cases}$$

It follows that the system has no solution (**inconsistent**).

EXAMPLE: Find all solutions of the system

$$\begin{cases} x + y = 1 \\ x + y = 1 \end{cases}$$

Answer: The system has infinitely many solutions (**dependent**).

EXAMPLE: Find all solutions of the system

$$\begin{cases} 15x - 10y = 25 \\ -24x + 16y = -40 \end{cases}$$

Solution: We have

$$\begin{cases} 15x - 10y = 25 \\ -24x + 16y = -40 \end{cases} \iff \begin{cases} 3x - 2y = 5 \\ 3x - 2y = 5 \end{cases} \iff \begin{cases} 3x - 2y = 5 \\ 0 = 0 \end{cases}$$

It follows that the system has infinitely many solutions (**dependent**).