

Name: _____ ID#: _____

Final Exam

Elementary Statistics

May 15, 2014

PLEASE READ THE FOLLOWING INFORMATION.

- This is a **110-minute** exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. The quality assurance engineer of a television manufacturer inspects TVs in lots of 100. He selects 5 of the 100 TVs at random and inspects them thoroughly. Assuming that 6 of the 100 TVs in the current lot are defective, find the probability that exactly 2 of the 5 TVs selected by the engineer are defective.

Solution: We apply the Hypergeometric Distribution formula

$$f(x) = \frac{\binom{a}{x} \cdot \binom{b}{n-x}}{\binom{a+b}{n}}$$

with $n = 5$, $a = 6$, $b = 94$, and $x = 2$. We have

$$f(2) = \frac{\binom{6}{2} \cdot \binom{94}{5-2}}{\binom{6+94}{5}} = \frac{\binom{6}{2} \cdot \binom{94}{3}}{\binom{100}{5}} = \frac{15 \cdot 134,044}{75,287,520} = \frac{2,010,660}{75,287,520} \approx 0.027$$

So, there is a 2.7% chance that exactly 2 of the 5 TVs selected by the engineer will be defective.

Info: The average in the class for this problem was 64.1%.

2. On New Year's Eve, the probability of a person having a car accident is 0.09. The probability of a person driving while intoxicated is 0.32 and probability of a person having a car accident while intoxicated is 0.15. What is the probability of a person driving while intoxicated or having a car accident?

Solution: Let I be the event that a person is driving while intoxicated and let A be the event that a person is having a car accident. Then

$$P(I \cup A) = P(I) + P(A) - P(I \cap A) = 0.32 + 0.09 - 0.15 = 0.26$$

Info: The average in the class for this problem was 88.1%.

3. In the paper “Persistent Pulmonary Hypertension of the Neonate and Asymmetric Growth Restriction” (Obstetrics & Gynecology, Vol. 91, No. 3, pp. 336-341), M. Williams et al. reported on a study of characteristics of neonates. Infants treated for pulmonary hypertension, called the PH group, were compared with those not so treated, called the control group. One of the characteristics measured was head circumference. The mean head circumference of the 10 infants in the PH group was 34.2 centimeters (cm). Assuming that head circumferences for infants treated for pulmonary hypertension are normally distributed with standard deviation 2.1 cm, determine the sample size required to have a margin of error of 0.5 cm with a 95% confidence level.

Solution: To find the sample size, we use the formula $n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$. We know that $\sigma = 2.1$ and $E = 0.5$. The confidence level is 0.95, which means that $\alpha = 0.05$ and $z_{\alpha/2} = z_{0.025} = 1.96$. Thus

$$n = \left(\frac{1.96 \cdot 2.1}{0.5}\right)^2 \approx 67.7658$$

which, rounded up to the nearest whole number, is 68.

Info: The average in the class for this problem was 63.7%.

4. For a given population with $\sigma = \$15$, we want to test the null hypothesis $\mu = \$80$ on the basis of random sample of size $n = 50$. If the null hypothesis is rejected when x is greater than or equal to $\$83$ and otherwise it is accepted, find
- the probability of Type I error;
 - the probability of Type II error when $\mu = \$82$;
 - the probability of Type II error when $\mu = \$84$.

Solution: First we note that $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{50}}$.

(a) We have

$$z = \frac{83 - 80}{15/\sqrt{50}} = \sqrt{2} \approx 1.41$$

and the probability is $0.5 - 0.4207 = 0.0793$.

(b) We have

$$z = \frac{83 - 82}{15/\sqrt{50}} = \frac{\sqrt{2}}{3} \approx 0.47$$

and the probability is $0.5 + 0.1808 = 0.6808$.

(c) We have

$$z = \frac{83 - 84}{15/\sqrt{50}} = -\frac{\sqrt{2}}{3} \approx -0.47$$

and the probability is $0.5 - 0.1808 = 0.3192$.

Info: The average in the class for this problem was 50.6%.

5. The table below provides a sample of 19 exam grades randomly selected from a large class.

76	74	82	96	66
76	78	72	52	68
86	84	62	76	78
92	82	74	88	

Construct a stem-and-leaf display.

Solution:

5	2						
6	2	6	8				
7	2	4	4	6	6	6	8 8
8	2	2	4	6	8		
9	2	6					

Info: The average in the class for this problem was 96.7%.

6. Consider the following data for shipments of peanuts from a hypothetical U.S. exporter to five Canadian cities.

City	Peanuts (Thousands of Bags)
Montreal	64.0
Ottawa	15.0
Toronto	285.0
Vancouver	228.0
Winnipeg	45.0

Let's assume that shipments to the respective cities will be sold at the following profits per thousand bags: \$15.00, \$13.50, \$15.50, \$12.00, and \$14.00. Find the average profit per thousand bags.

Solution: By the formula for the weighted mean, we get

$$\frac{(64)(\$15.0) + (15)(\$13.5) + (285)(\$15.5) + (228)(\$12.0) + (45)(\$14.0)}{64 + 15 + 285 + 228 + 45} = \frac{\$8946}{637} \approx \$14.04$$

Info: The average in the class for this problem was 87.3%.

7. An urn contains 6 red marbles and 4 black marbles. Two marbles are drawn without replacement from the urn. What is the probability that both of the marbles are black?

Solution: Let B_1 and B_2 be the events that the first and second marbles are black, respectively. Then we want $P(B_1B_2)$. Note that we easily have that $P(B_1) = 4/10$. Also, $P(B_2|B_1) = 3/9$ because the *reduced* sample space B_1 consists of 3 black marbles and 6 red marbles, and we are asking for the probability of choosing a black marble. Therefore,

$$P(B_1B_2) = P(B_1)P(B_2|B_1) = \frac{4}{10} \cdot \frac{3}{9} = \frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}$$

Note, that we could also do it straightaway as $\binom{4}{2} / \binom{10}{2}$.

Info: The average in the class for this problem was 79.4%.

8. In 1908, W. S. Gosset published the article “The Probable Error of a Mean” (Biometrika, Vol. 6, pp. 125). In this pioneering paper, written under the pseudonym “Student,” Gosset introduced what later became known as Students t -distribution. Gosset used the following data set, which gives the additional sleep in hours obtained by a sample of 10 patients using laevohysocyanine hydrobromide.

1.9	0.8	1.1	0.1	-0.1
4.4	5.5	1.6	4.6	3.4

- (a) Find the mean of the sample.

Solution: We have

$$\bar{x} = \frac{1.9 + 0.8 + 1.1 + 0.1 + (-0.1) + 4.4 + 5.5 + 1.6 + 4.6 + 3.4}{10} = \frac{23.3}{10} = 2.33 \text{ hr}$$

- (b) Find the standard deviation of the sample.

Solution: We will use the formula

$$s = \sqrt{\frac{S_{xx}}{n-1}}, \quad \text{where} \quad S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

First we calculate $\sum x$ and $\sum x^2$, getting

$$\sum x = 1.9 + 0.8 + 1.1 + 0.1 + (-0.1) + 4.4 + 5.5 + 1.6 + 4.6 + 3.4 = 23.3$$

and

$$\sum x^2 = 1.9^2 + 0.8^2 + 1.1^2 + 0.1^2 + (-0.1)^2 + 4.4^2 + 5.5^2 + 1.6^2 + 4.6^2 + 3.4^2 = 90.37$$

Therefore

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 90.37 - \frac{23.3^2}{10} \approx 36.08$$

hence

$$s = \sqrt{\frac{S_{xx}}{n-1}} = \sqrt{\frac{36.08}{9}} \approx 2.002 \text{ hr}$$

(c) Obtain a 95% confidence interval for the additional sleep that would be obtained on average for all people using laevohydroxyamphetamine hydrobromide.

Solution: We want a 95% confidence interval, so $\alpha = 1 - 0.95 = 0.05$. For $n = 10$, we have $df = 10 - 1 = 9$. From Table II,

$$t_{\alpha/2} = t_{0.05/2} = t_{0.025} = 2.262$$

The confidence interval for μ is from

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{to} \quad \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}.$$

We know that $\bar{x} = 2.33$ and $s = 2.002$, therefore a 95% confidence interval for μ is from

$$2.33 - 2.262 \cdot \frac{2.002}{\sqrt{10}} \quad \text{to} \quad 2.33 + 2.262 \cdot \frac{2.002}{\sqrt{10}}$$

or 0.90 to 3.76 hr.

(d) Interpret your answer from part (c).

Solution: It appears so because, based on the confidence interval, we can be 95% confident that the mean additional sleep is somewhere between 0.90 hr and 3.76 hr and that, in particular, the mean is positive.

Info: The average in the class for this problem was 73.1%.

9. Researchers G. Evans and E. Kantrowitz explored the health consequences for exposure to many different environmental risks in the journal article “Socioeconomic Status and Health: The Potential Role of Environmental Risk Exposure” (Annual Review of Public Health, Vol. 23, No. 1, pp. 303-331). According to research, 65% of preschool children living in poverty have been exposed to cigarette smoke at home. If 200 preschool children living in poverty are selected at random, what is the probability that at least 125 have been exposed to cigarette smoke at home?

Solution: We have $n = 200$ and $p = 0.65$, therefore

$$np = 200 \cdot 0.65 = 130 \quad \text{and} \quad n(1 - p) = 200 \cdot 0.35 = 70$$

Both np and $n(1 - p)$ are greater than 5, so we can approximate the binomial distribution by normal. We get

$$\mu = np = 130 \quad \text{and} \quad \sigma = \sqrt{np(1 - p)} = \sqrt{200 \cdot 0.65 \cdot 0.35} \approx 6.745$$

To make the correction for continuity, we subtract 0.5 from 125. Thus we need to find the area under the normal curve with parameters $\mu = 130$ and $\sigma = 6.745$ that lies to the right of 124.5. We have

$$z = \frac{124.5 - 130}{6.745} \approx -0.82$$

therefore the probability is about $0.2939 + 0.5 = 0.7939$ that least 125 have been exposed to cigarette smoke at home.

Info: The average in the class for this problem was 42.9%.

10. A saleswoman has been offered a new job with a fixed salary of \$290. Her records from her present job show that her weekly commissions have the following probabilities:

Commission	0	\$100	\$200	\$300	\$400
Probability	0.05	0.15	0.25	0.45	0.1

Should she change her present job?

Solution: The expected value from her present job is

$$(0 \cdot \$0.05) + (0.15 \cdot \$100) + (0.25 \cdot \$200) + (0.45 \cdot \$300) + (0.1 \cdot \$400) = \$240$$

Since $\$290 > \240 , she should change her present job.

Info: The average in the class for this problem was 87.6%.

11. Data on investments in the high-tech industry by venture capitalists are compiled by VentureOne Corporation and published in America's Network Telecom Investor Supplement. A random sample of 18 venturecapital investments in the fiber optics business sector yielded the following data, in millions of dollars.

5.60	6.27	5.96	10.51	2.04	5.48
5.74	5.58	4.13	8.63	5.95	6.67
4.21	7.71	9.21	4.98	8.64	6.66

Determine a 95% confidence interval for the mean amount, μ , of all venture-capital investments in the fiber optics business sector. Assume that the population standard deviation is \$2.04 million. (Note: The sum of the data is \$113.97 million.)

Solution: From Table I, $z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$. The confidence interval for μ is from

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{to} \quad \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

We know $\sigma = 2.04$, $n = 18$, and $z_{\alpha/2} = 1.96$. To compute \bar{x} for the data in the Table above, we apply the usual formula:

$$\bar{x} = \frac{\sum x_i}{n} = \frac{113.97}{18} \approx 6.332$$

to one decimal place. Consequently, a 95% confidence interval for μ is from

$$6.332 - 1.96 \cdot \frac{2.04}{\sqrt{18}} \quad \text{to} \quad 6.332 + 1.96 \cdot \frac{2.04}{\sqrt{18}}$$

or \$5.389 million to \$7.274 million.

Info: The average in the class for this problem was 76.7%.

12. The scores achieved by students in America make the news often, and all kinds of conclusions are drawn based on these scores. The ACT Assessment is designed to assess high school students' general educational development and their ability to complete college-level work. One of the categories tested is Science Reasoning. The mean ACT test score for all high school graduates in 2008 in Science Reasoning was 20.8 with a standard deviation of 4.6. According to Chebyshev's theorem, at least what percent of high school graduates' ACT scores in Science Reasoning were between 11.6 and 30.0?

Solution: Since

$$30.0 - 20.8 = 20.8 - 11.6 = 9.2$$

it follows that

$$k = \frac{9.2}{4.6} = 2$$

therefore at least

$$1 - \frac{1}{2^2} = \frac{3}{4} = 75\%$$

of high school graduates' ACT scores in Science Reasoning were between 11.6 and 30.0.

Info: The average in the class for this problem was 78.7%.

13. A variable is normally distributed with mean 6 and standard deviation 2. Find the percentage of all possible values of the variable that
(a) lie between 1 and 7.

Solution: We have

$$z_1 = \frac{1 - 6}{2} = -2.5 \quad \text{and} \quad z_2 = \frac{7 - 6}{2} = 0.5$$

therefore the probability is $0.4938 + 0.1915 = 0.6853$, so the percentage is 68.53%.

- (b) exceed 5.

Solution: We have

$$z = \frac{5 - 6}{2} = -0.5$$

therefore the probability is $0.1915 + 0.5 = 0.6915$, so the percentage is 69.15%.

- (c) are less than 4.

Solution: We have

$$z = \frac{4 - 6}{2} = -1$$

therefore the probability is $0.5 - 0.3413 = 0.1587$, so the percentage is 15.87%.

Info: The average in the class for this problem was 75.6%.

14. If ice cream sundaes come in 5 flavors with 4 possible toppings, how many different sundaes can be made with one flavor of ice cream and one topping?

Solution: By the Basic Principle of Counting, the number of different sundaes is $5 \cdot 4 = 20$.

Info: The average in the class for this problem was 96%.

15. Scores on the SAT test have a mean of 1518 and a standard deviation of 325. Scores on the ACT test have a mean of 21.1 and a standard deviation of 4.8. Which is relatively better: a score of 1840 on the SAT test or a score of 26.0 on the ACT test? Why?

Solution: We have

$$z_1 = \frac{1840 - 1518}{325} \approx 0.99 \quad \text{and} \quad z_2 = \frac{26.0 - 21.1}{4.8} \approx 1.02$$

The score of 26.0 is relatively better because it has the larger z score.

Info: The average in the class for this problem was 88.9%.

16. A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Solution: $0.25/0.42=0.6$

Info: The average in the class for this problem was 79.8%.

17. In how many ways can the letters of the word “statistics” be arranged?

Solution: We have

$$\frac{10!}{3!3!2!} = 50400$$

Info: The average in the class for this problem was 80.2%.

TABLE A.1 AREAS OF THE NORMAL DISTRIBUTION

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
35	1.306	1.690	2.030	2.438	2.724	35
40	1.303	1.684	2.021	2.423	2.704	40
50	1.299	1.676	2.009	2.403	2.678	50
60	1.296	1.671	2.000	2.390	2.660	60
70	1.294	1.667	1.994	2.381	2.648	70
80	1.292	1.664	1.990	2.374	2.639	80
90	1.291	1.662	1.987	2.369	2.632	90
100	1.290	1.660	1.984	2.364	2.626	100
1000	1.282	1.646	1.962	2.330	2.581	1000
2000	1.282	1.646	1.961	2.328	2.578	2000