Name: ______________________________ ID#: ______________

Midterm Exam

Elementary Statistics

October 17, 2013

PLEASE READ THE FOLLOWING INFORMATION.

• This is a **75-minute** exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.

• You are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. The Television Bureau of Advertising publishes information on television ownership in Trends in Television. The Table below gives the number of TV sets per household for 50 randomly selected households.

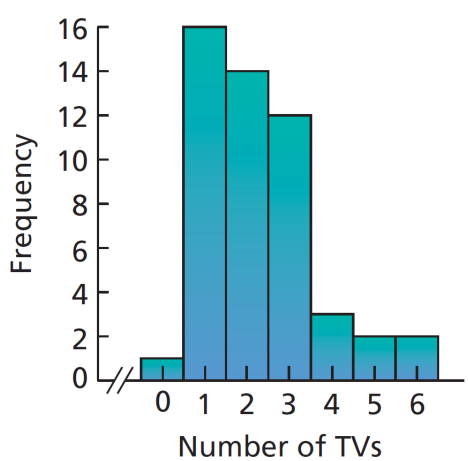
1	1	1	2	6	3	3	4	2	4
3	2	1	5	2	1	3	6	2	2
3	1	1	4	3	2	2	2	2	3
0	3	1	2	1	2	3	1	1	3
3	2	1	2	1	1	3	1	5	1

(a) Organize these data into a frequency distribution, percentage distribution and cumulative distribution.

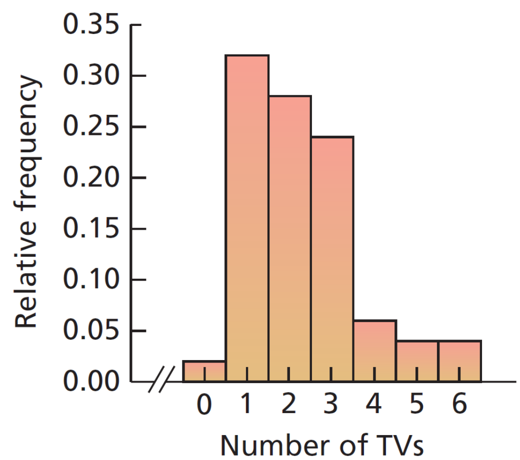
Number of TV's	Frequency	Percentage
0	1	0.02
1	16	0.32
2	14	0.28
3	12	0.24
4	3	0.06
5	2	0.04
6	2	0.04

Number of TV's	Cumulative Frequency
less than 1	1
less than 2	17
less than 3	31
less than 4	43
less than 5	46
less than 6	48
less than 7	50

(b) Draw its frequency histogram.



or



2. The Table below displays the number of days to maturity for 40 short-term investments.

70	64	99	55	64	89	87	65
62	38	67	70	60	69	78	39
75	56	71	51	99	68	95	86
57	53	47	50	55	81	80	98
51	36	63	66	85	79	83	70

Construct a stem-and-leaf display with the stem labels 3, 4, 5, 6, 7, 8, 9.

Stem	Leaves
3	8 9 6
4	7
5	7 1 6 3 5 1 0 5
6	2 4 7 3 6 4 0 9 8 5
7	0 5 1 0 9 8 0
8	5 9 1 7 0 3 6
9	9 9 5 8

or

Stem	Leaves
3	6 8 9
4	7
5	0 1 1 3 5 5 6 7
6	0 2 3 4 4 5 6 7 8 9
7	0 0 0 1 5 8 9
8	0 1 3 5 6 7 9
9	5 8 9 9

3. A home appliance center advertised the following refrigerators, of which it had, respectively, 18, 12, 9, 14, and 25 in stock.

Brand	Size	Price
Company A	15 cu. ft.	\$416
Company B	21 cu. ft.	\$549
Company C	19 cu. ft.	\$649
Company D	21 cu. ft.	\$716
Company E	24 cu. ft.	\$799

(a) What is the average size of these refrigerators?

$$\overline{x}_w = \frac{18(15) + 12(21) + 9(19) + 14(21) + 25(24)}{18 + 12 + 9 + 14 + 25} = \frac{1,587}{78} = 20.35 \text{ cubic feet}$$

(b) What is the average price of these refrigerators?

$$\overline{x}_w = \frac{18(416) + 12(549) + 9(649) + 14(716) + 25(799)}{18 + 12 + 9 + 14 + 25} = \frac{49,916}{78} = \$639.95$$

4. Following are fourteen temperature readings taken at different locations in a large kiln: 409, 412, 439, 411, 432, 432, 405, 411, 422, 417, 440, 427, 411, and 417. Find the median, Q_1 and Q_3.

   Solution: We first arrange the data in ascending order:

   405, 409, 411, 411, 411, 412, 417, 417, 422, 427, 432, 432, 439, 440

   It follows that the median is 417, Q_1 is 411, and Q_3 is 432.

5. Following are the numbers of blossoms on 50 cacti in a desert botanical garden: 1, 0, 3, 0, 4, 1, 0, 1, 0, 0, 1, 6, 1, 0, 0, 0, 3, 3, 0, 1, 1, 5, 0, 2, 0, 3, 1, 1, 0, 4, 0, 0, 1, 2, 1, 1, 2, 0, 1, 0, 3, 0, 0, 1, 5, 3, 0, 0, 1, and 0. Find the mode.

   Solution: The mode is 0, which occurs 21 times.

6. Having kept records for several months, Ms. Lewis knows that it takes her on the average 47.7 minutes with a standard deviation of 2.46 minutes to drive to work from her suburban home. If she always starts out exactly one hour before she has to arrive at work, at most what percent of the time will she arrive late?

   Solution: We will apply Chebyshev's theorem. Since $60 - 47.7 = 12.3$, we find that $k(2.46) = 12.3$ and, hence,

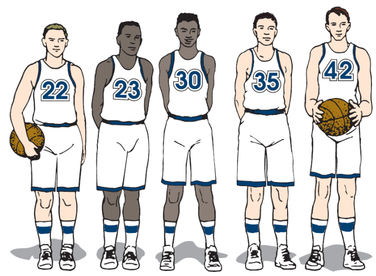
   $$k = \frac{12.3}{2.46} = 5$$

   It follows that at least

   $$1 - \frac{1}{5^2} = 1 - \frac{1}{25} = \frac{24}{25} = 0.96$$

   So at least 96% of time she arrives on time, therefore she is late at most 4% of the time.

7. The heights of the five starting players on a men's college basketball team are shown in the Figure below.



Feet and inches	6'	6'1"	6'4"	6'4"	6'6"
Inches	72	73	76	76	78

(a) Find the range.

$$\text{Range} = 78 - 72 = 6 \text{ inches}$$

(b) Find the mean.

$$\text{Mean} = \frac{72 + 73 + 76 + 76 + 78}{5} = \frac{375}{5} = 75 \text{ inches}$$

(c) Find the standard deviation.

Solution: Since

$$72 + 73 + 76 + 76 + 78 = 375$$

and

$$72^2 + 73^2 + 76^2 + 76^2 + 78^2 = 28,149$$

it follows that

$$s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}} = \sqrt{\frac{28,149 - \frac{375^2}{5}}{5-1}} = \sqrt{\frac{28,149 - 28,125}{4}} = \sqrt{\frac{24}{4}} = \sqrt{6} \approx 2.4 \text{ inches}$$

8. In how many different ways can the judges choose a winner and first runner-up from the 10 finalists in a student essay contest?

$$10 \cdot 9 = 90$$

9. An insurance company has a branch in each of the six New England states (Massachusetts, Rhode Island, Connecticut, Maine, New Hampshire, and Vermont). Find the number of ways in which six branch managers can be assigned to manage an office in each of the six states.

$$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$$

10. A motel chain wants to inspect 5 of its 32 franchised operations. If the order of the inspections does not matter, in how many different ways can it plan this series of inspections?

$$\binom{32}{5} = \frac{32!}{5!(32-5)!} = \frac{32!}{5!27!} = \frac{27! \cdot 28 \cdot 29 \cdot 30 \cdot 31 \cdot 32}{5!27!} = \frac{28 \cdot 29 \cdot 30 \cdot 31 \cdot 32}{5!} = 201,376$$

11. Let $A = \{3,4\}$, $B = \{1,2,3,4,5,6,7\}$, and $C = \{6,7,8\}$ be subsets of the sample space $S = \{1,2,3,4,5,6,7,8\}$. Find

(a) $B' = \{8\}$

(b) $A \cap B = \{3,4\}$

(c) $B \cap C' = \{1,2,3,4,5,6,7\} \cap \{1,2,3,4,5\} = \{1,2,3,4,5\}$

(d) $A \cup C = \{3,4,6,7,8\}$

(e) $A \cup B' = \{3,4\} \cup \{8\} = \{3,4,8\}$

12. A carton of 24 light bulbs includes two that are defective. If two of the bulbs are chosen at random, what are the probabilities that

(a) neither bulb will be defective;

Solution 1: Let N_1 and N_2 be the events that the first and second bulb are not defective, respectively. Then we want $P(N_1N_2)$. Note that we easily have that $P(N_1) = 22/24$. Also, $P(N_2|N_1) = 21/23$ because after a non-defective bulb is chosen, the total number of bulbs is 23 and the number of non-defective bulbs is 21. Therefore,

$$P(N_1N_2) = P(N_1)P(N_2|N_1) = \frac{22}{24} \cdot \frac{21}{23} = \frac{462}{552} = \frac{77}{92} \approx 0.84$$

Solution 2: We have

$$\frac{\binom{22}{2}}{\binom{24}{2}} = \frac{\frac{22!}{2!(22-2)!}}{\frac{24!}{2!(24-2)!}} = \frac{\frac{22!}{2!20!}}{\frac{24!}{2!22!}} = \frac{\frac{20!\cdot 21\cdot 22}{2!20!}}{\frac{22!\cdot 23\cdot 24}{2!22!}} = \frac{\frac{21\cdot 22}{2!}}{\frac{23\cdot 24}{2!}} = \frac{21\cdot 22}{23\cdot 24} = \frac{462}{552} = \frac{77}{92} \approx 0.84$$

(b) one of the bulbs will be defective;

Solution 1: Let N_1 and N_2 be the events that the first and second bulb are not defective, respectively. Let also D_1 and D_2 be the events that the first and second bulb are defective, respectively. Then we want $P(N_1D_2 \cup D_1N_2)$. Note that we easily have that $P(N_1) = 22/24$. Also, $P(D_2|N_1) = 22/23$ because after a non-defective bulb is chosen, the total number of bulbs is 23 and the number of defective bulbs is still 2. Therefore,

$$P(N_1D_2) = P(N_1)P(D_2|N_1) = \frac{22}{24} \cdot \frac{2}{23}$$

Similarly,

$$P(D_1N_2) = P(D_1)P(N_2|D_1) = \frac{2}{24} \cdot \frac{22}{23}$$

Therefore

$$P(N_1D_2 \cup D_1N_2) = \frac{22}{24} \cdot \frac{2}{23} + \frac{2}{24} \cdot \frac{22}{23} = \frac{22\cdot 2}{24\cdot 23} + \frac{22\cdot 2}{24\cdot 23} = 2 \cdot \frac{22\cdot 2}{24\cdot 23} = \frac{88}{552} = \frac{11}{69} \approx 0.16$$

Solution 2: We have

$$\frac{\binom{2}{1}\binom{22}{1}}{\binom{24}{2}} = \frac{2\cdot 22}{\frac{24!}{2!(24-2)!}} = \frac{2\cdot 22}{\frac{24!}{2!22!}} = \frac{2\cdot 22}{\frac{22!\cdot 23\cdot 24}{2!22!}} = \frac{2\cdot 22}{\frac{23\cdot 24}{2!}} = \frac{2\cdot 2\cdot 22}{23\cdot 24} = \frac{88}{552} = \frac{11}{69} \approx 0.16$$

(c) both bulbs will be defective?

Solution 1: Let D_1 and D_2 be the events that the first and second bulb are defective, respectively. Then we want $P(D_1 D_2)$. Note that we easily have that $P(D_1) = 2/24$. Also, $P(D_2|D_1) = 1/23$ because after a defective bulb is chosen, the total number of bulbs is 23 and the number of defective bulbs is 1. Therefore,

$$P(D_1 D_2) = P(D_1)P(D_2|D_1) = \frac{2}{24} \cdot \frac{1}{23} = \frac{2}{552} = \frac{1}{276} \approx 0.0036$$

Solution 2: We have

$$\frac{\binom{2}{2}}{\binom{24}{2}} = \frac{1}{\frac{24!}{2!(24-2)!}} = \frac{1}{\frac{24!}{2!22!}} = \frac{1}{\frac{22! \cdot 23 \cdot 24}{2!22!}} = \frac{1}{\frac{23 \cdot 24}{2!}} = \frac{2}{23 \cdot 24} = \frac{2}{552} = \frac{1}{276} \approx 0.0036$$

13. An auction house has two appraisers of precious jewelry. The probability that the older of the two will not be available is 0.33, the probability that the other one will not be available is 0.27, and the probability that both of them will not be available is 0.19. What is the probability that either or both of them will not be available?

$$0.33 + 0.27 - 0.19 = 0.41$$

14. The probability that a bus from Seattle to Vancouver will leave on time is 0.80, and the probability that it will leave on time and also arrive on time is 0.72.

(a) What is the probability that a bus that leaves on time will also arrive on time?

$$\frac{0.72}{0.80} = 0.90$$

(b) If the probability that such a bus will arrive on time is 0.75, what is the probability that a bus that arrives on time also left on time?

$$\frac{0.72}{0.75} = 0.96$$

15. How many different 5-letter codes can be made using three a's and two b's?

$$\frac{5!}{3!2!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{(1 \cdot 2 \cdot 3)(1 \cdot 2)} = \frac{4 \cdot 5}{1 \cdot 2} = \frac{20}{2} = 10$$