Name: ______________________________ ID#: ______________

Final Exam

Elementary Statistics

December 19, 2013

PLEASE READ THE FOLLOWING INFORMATION.

• This is a **110-minute** exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.

• You are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. It is desired to estimate the average number of hours of continuous use until a model 737 airplane will first require repairs. If it can be assumed that $\sigma = 138$ hours for such data, how large a sample is needed to be able to assert with a probability of 0.99 that the sample mean will be off by no more than 40 hours?

   Solution: $n = \left(\frac{2.575 \cdot 138}{40}\right)^2 = 78.92$ and $n = 79$ rounded up to the nearest integer.

2. While performing a certain task under simulated weightlessness, the pulse rate of 12 astronauts increased on the average by 27.33 beats per minute with a standard deviation of 4.28 beats per minute. Construct a 99% confidence interval for the true average increase in the pulse rate of astronauts performing the given task (under the stated condition).

   Solution: Substituting $n = 12$, $\overline{x} = 27.33$, $s = 4.28$, and $t_{0.005} = 3.106$ (the entry in Table II for $12 - 1 = 11$ degrees of freedom) into the t interval formula, we get

$$27.33 - 3.106 \cdot \frac{4.28}{\sqrt{12}} < \mu < 27.33 + 3.106 \cdot \frac{4.28}{\sqrt{12}}$$

   and, hence,

$$23.49 < \mu < 31.17$$

   beats per minute.

3. How many 3 digit numbers can you make using the digits $1, 2, 3, \ldots, 9$ without repeating the digits?

   Solution: $N = 9 \cdot 8 \cdot 7 = 504$

4. Suppose two students of the class of 8 refuse to be on the same team. How many distinct 4-person teams can be formed?

   Solution: $N = \binom{8}{4} - \binom{6}{2} = 70 - \frac{6!}{2!4!} = 70 - \frac{5 \cdot 6}{2!} = 70 - 15 = 55$

5. How many different 7-letter codes can be made using three a's and four b's?

   Solution: $\frac{7!}{3!4!} = \frac{5 \cdot 6 \cdot 7}{3!} = \frac{5 \cdot 7}{1} = 35$

6. A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

   Solution: $P(E) = 3/12 = 1/4$

7. The response times for a sample of 6 switches designed to activate an alarm system upon receiving a certain stimulus are $9, 8, 5, 11, 7,$ and 5 milliseconds. Find the mean, sample standard deviation and sample variance.

   Solution: Using the computing formula we first calculate $\sum x$ and $\sum x^2$ getting

$$\sum x = 9 + 8 + 5 + 11 + 7 + 5 = 45 \quad \text{and} \quad \sum x^2 = 9^2 + 8^2 + 5^2 + 11^2 + 7^2 + 5^2 = 365$$

   It follows that

$$\overline{x} = \frac{45}{5} = 9$$

   To find s, we first substitute the values of $\sum x$ and $\sum x^2$ into the formula for S_{xx}:

$$S_{xx} = 365 - \frac{45^2}{6} = 27.5$$

   Therefore

$$s = \sqrt{\frac{27.5}{6-1}} = \sqrt{\frac{27.5}{5}} = \sqrt{5.5} = 2.3$$

   and

$$s^2 = 5.5$$

8. Mrs. Clark belongs to an age group for which the mean weight is 112 pounds with a standard deviation of 11 pounds, and Mr. Clark, her husband, belongs to an age group for which the mean weight is 163 pounds with a standard deviation of 18 pounds. If Mrs. Clark weighs 132 pounds and Mr. Clark weighs 193 pounds, which of the two is relatively more overweight compared to his/her age group?

   Solution: Mr. Clark's weight is $193 - 163 = 30$ pounds above average while Mrs. Clark's weight is "only" $132 - 112 = 20$ pounds above average, yet in standard units we get $\frac{193-163}{18} \approx 1.67$ for Mr. Clark and $\frac{132-112}{11} \approx 1.82$ for Mrs. Clark. Thus, relative to them age groups Mrs. Clark is somewhat more overweight than Mr. Clark.

9. If the mean of 10 nonnegative numbers is 100, can

(a) three of these numbers be > 330? Why?

Solution: Yes. For example,

$$n_1 = 331,\ n_2 = 331,\ n_3 = 331,\ n_4 = n_5 = \ldots = n_{10} = 1$$

(b) three of these numbers be > 350? Why?

Solution: No. Indeed, if three of these numbers are > 350, then the sum of these numbers is $> 350 \cdot 3 = 1050$. We get a contradiction, since no part, or subset of the data can exceed

$$n \cdot \overline{x} = 10 \cdot 100 = 1000$$

(c) all these numbers be < 110? Why?

Solution: Yes. For example,

$$n_1 = n_2 = n_3 = \ldots = n_{10} = 100$$

(d) all these numbers be < 90? Why?

Solution: No. Indeed, if all these numbers are < 90, then the sum of these numbers is

$$< 10 \cdot 90 = 900$$

and therefore the mean is

$$< \frac{900}{10} = 90$$

We get a contradiction, since the mean is 100.

(e) all these numbers be > 90? Why?

Solution: Yes. For example,

$$n_1 = n_2 = n_3 = \ldots = n_{10} = 100$$

10. Two dice are rolled. We define events E1, E2, E3 and E4 as follows

    E1: Getting a sum equal to 10.

    E2: Getting a double.

    E3: Getting a sum less than 4.

    E4: Getting a sum less than 7.

    (a) Are events E1 and E2 exclusive? Why?

    Solution: E1 and E2 are not exclusive because outcome (5,5) is a double and also gives a sum of 10. The two events may occur at the same time.

    (b) Are events E2 and E3 exclusive? Why?

    Solution: E2 and E3 are not exclusive because outcome (1,1) is a double and gives a sum of 2 and is less than 4. The two events E2 and E3 may occur at the same time.

    (c) Are events E3 and E4 exclusive? Why?

    Solution: E3 and E4 are not exclusive a sum can be less than 7 and less than 4 a the same time. Example outcome (1,2).

    (d) Are events E4 and E1 exclusive? Why?

    Solution: E4 and E1 are exclusive because a sum less than 7 cannot be equal to 10 at the same time. The two events cannot occur at the same time.

11. An auction house has two appraisers of precious jewelry. The probability that the older of the two will not be available is 0.33, the probability that the other one will not be available is 0.27, and the probability that both of them will not be available is 0.19. What is the probability that either or both of them will not be available?

$$0.33 + 0.27 - 0.19 = 0.41$$

12. The probability that Henry will like a new movie is 0.70 and the probability that Jean, his girlfriend, will like it is 0.60. If the probability is 0.28 that he will like it and she will dislike it, what is the probability that he will like it given that she is not going to like it?

Solution: If H and J are the events that Henry will like the new movie and that Jean will like it, we have

$$P(J') = 1 - 0.60 = 0.40 \quad \text{and} \quad P(H \cap J') = 0.28$$

therefore

$$P(H|J') = \frac{P(H \cap J')}{P(J')} = \frac{0.28}{0.40} = 0.70$$

13. Calculate the probability that four of six hibiscus plants will fail to survive a frost if the probability is 0.30 that any such plant will survive a frost.

Solution: We have

$$\binom{6}{4}(0.70)^4(0.30)^2 = 0.3241$$

14. A study shows that 60% of all first-class letters between any two cities in Ohio are delivered within 48 hours. Find the mean and the variance of the number of such letters, among eight, that are delivered within 48 hours.

Solution: We have

$$\mu = 8(0.60) = 4.80$$

and

$$\sigma^2 = 8(0.60)(0.40) = 1.92$$

15. Birth weights of babies in Guatemala were found to have mean $\mu = 2860$ grams and standard deviation $\sigma = 440$ grams. Assuming that the birth weights of babies are normally distributed and a sample of 50 babies is to be selected

(a) What is the probability that the sample mean will be between 2740 and 2900 grams?

Solution: We have

$$z_1 = \frac{2740 - 2860}{440} \approx -0.27 \quad \text{and} \quad z_2 = \frac{2900 - 2860}{440} \approx 0.09$$

therefore the probability is $0.1064 + 0.0359 = 0.1423$.

(b) What is the probability that the sample mean will be larger than 3000 grams?

Solution: We have

$$z = \frac{3000 - 2860}{440} \approx 0.32$$

therefore the probability is $0.5 - 0.1255 = 0.3745$.

16. The probability is 0.26 that a cloud seeded with silver iodide will fail to show spectacular growth. Find the probability that among 30 clouds with silver iodide

(a) at most 8 will fail to show spectacular growth;

Solution: We have $\mu = 30(0.26) = 7.8$ and $\sigma = \sqrt{30(0.26)(0.74)} = \sqrt{5.772} = 2.40$. Therefore

$$\frac{8.5 - 7.8}{2.40} = 0.29$$

and the probability is $0.114 + 0.5 = 0.614$ rounded to three decimal places.

(b) only 8 will fail to show spectacular growth.

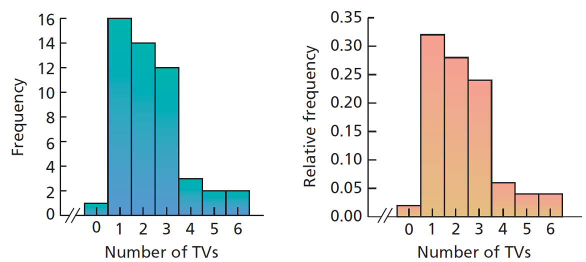
Solution: We have

$$\frac{7.5 - 7.8}{2.40} = -0.125 \quad \text{and} \quad \frac{8.5 - 7.8}{2.40} = 0.29$$

and the probability is about $0.05 + 0.114 = 0.16$ rounded to two decimal places.

17. The Television Bureau of Advertising publishes information on television ownership in Trends in Television. The Table below gives the number of TV sets per household for 50 randomly selected households. Organize these data into frequency and relative-frequency (percentage) distributions. Then construct a frequency histogram.

1	1	1	2	6	3	3	4	2	4
3	2	1	5	2	1	3	6	2	2
3	1	1	4	3	2	2	2	2	3
0	3	1	2	1	2	3	1	1	3
3	2	1	2	1	1	3	1	5	1



(a) (b)

18. Professor Hassett spent one summer working for a small mathematical consulting firm. The firm employed a few consultants, who made between \$300 and \$1050 per week. The Table below lists typical weekly earnings of employees. Find the range, mean, median and mode of the following data set:

\$300	300	300	940	300
300	400	300	400	
450	800	450	1050	

Solution: We have

$$\text{Range} = 1050 - 300 = 750$$

Mean

$$= \frac{300+300+300+940+300+300+400+300+400+450+800+450+1050}{13} \approx 484$$

$$\text{Median} = 400$$

$$\text{Mode} = 300$$

19. What is the probability of each possible sample if a random sample of size $n = 6$ is drawn from a finite population of size $N = 10$?

Solution: We have

$$\frac{1}{\binom{10}{6}} = \frac{1}{210}$$

20. In a study of automobile collision insurance costs, a random sample of $n = 35$ repair costs of front-end damage caused by hitting a wall at a specified speed had a mean of $1,438.

(a) Given that $\sigma = \$269$ for such data, what can be said with 98% confidence about the maximum error if $\overline{x} = \$1,438$ is used as an estimate of the average cost of such repairs?

Solution: We have

$$2.33 \cdot \frac{269}{\sqrt{35}} = \$106$$

(b) Construct a 90% confidence interval for the average cost of such repairs.

Solution: We have

$$1,438 - 1.645 \cdot \frac{269}{\sqrt{35}} < \mu < 1,438 + 1.645 \cdot \frac{269}{\sqrt{35}}$$

which gives

$$1363.2 < \mu < 1512.8$$

TABLE A.1 AREAS OF THE NORMAL DISTRIBUTION

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997

df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
35	1.306	1.690	2.030	2.438	2.724	35
40	1.303	1.684	2.021	2.423	2.704	40
50	1.299	1.676	2.009	2.403	2.678	50
60	1.296	1.671	2.000	2.390	2.660	60
70	1.294	1.667	1.994	2.381	2.648	70
80	1.292	1.664	1.990	2.374	2.639	80
90	1.291	1.662	1.987	2.369	2.632	90
100	1.290	1.660	1.984	2.364	2.626	100
1000	1.282	1.646	1.962	2.330	2.581	1000
2000	1.282	1.646	1.961	2.328	2.578	2000